Physical models for micro and nanosystems

Chapter 5: Micro- and nanoelectromechanical systems

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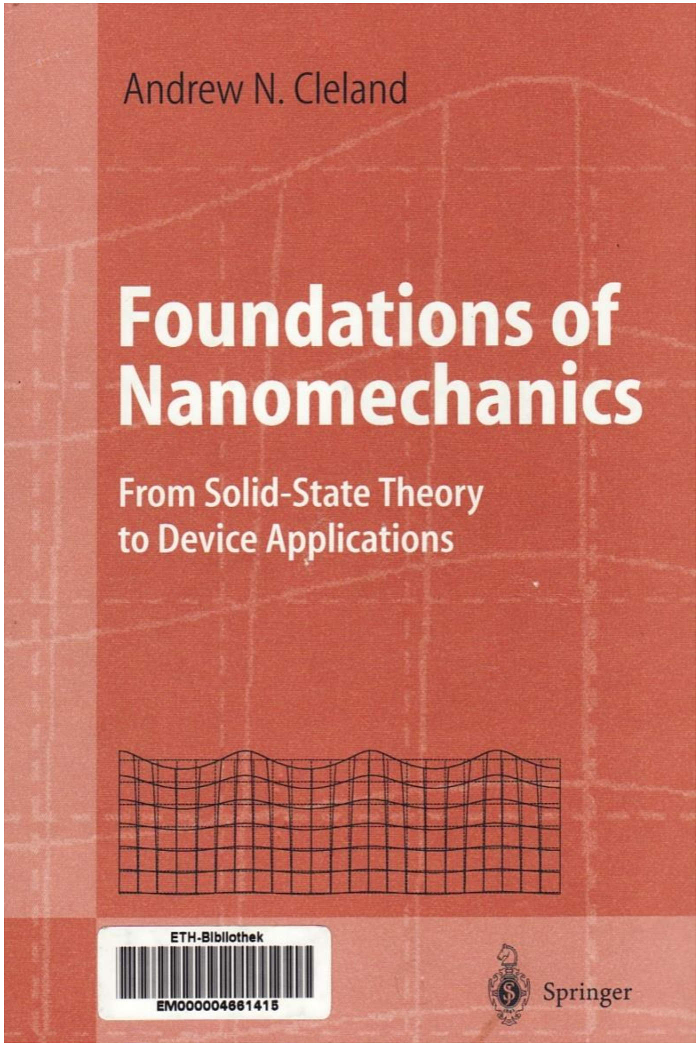
Reference

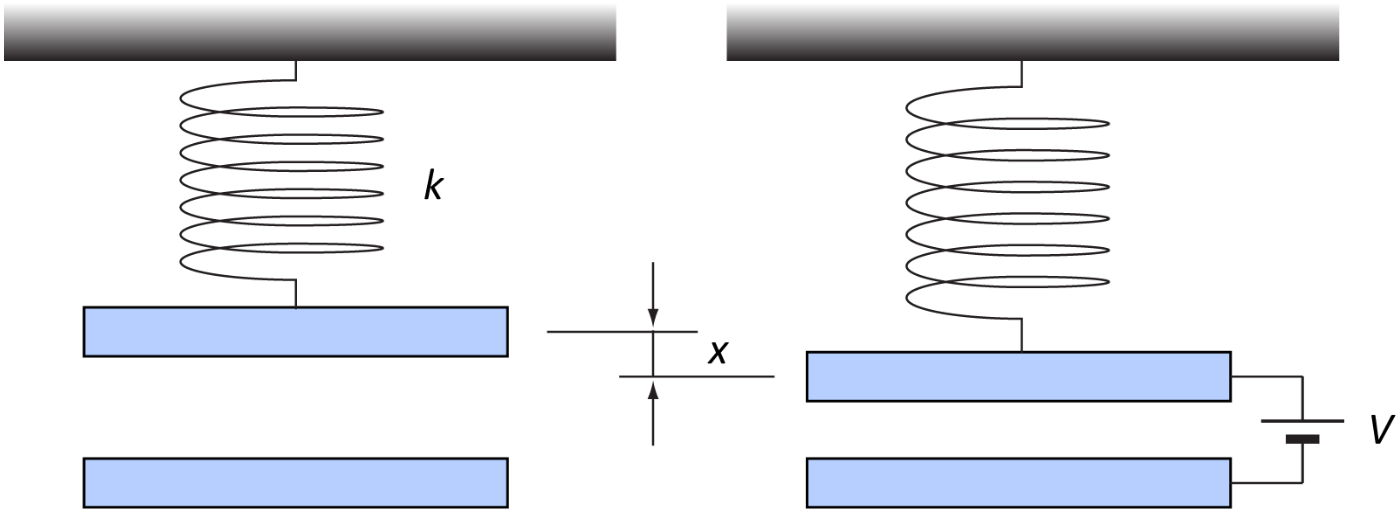
Foundations of Nanomechanics

Andrew N. Cleland



Simplest MEMS/NEMS

- Mechanical element deforms under an electrostatic force



- The deformation of the mechanical element can usually be reduced to one of these types of deformation:
  - stretching
  - bending
  - torsion

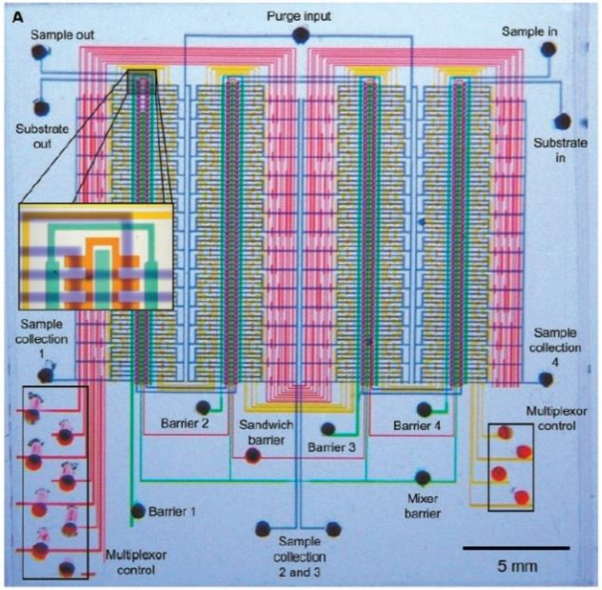
Examples of MEMS and NEMS devices

What are they for?

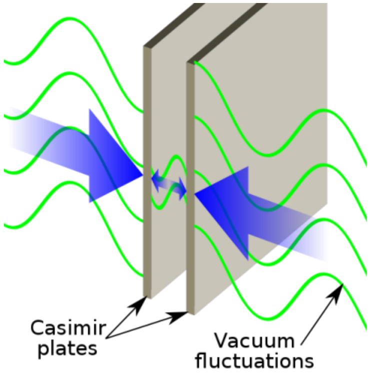
- Practical applications:
  - force/acceleration/orientation sensing
  - mass spectrometry on a chip – recognize atoms and molecules by weighing them
  - replace quartz oscillators with MEMS/NEMS

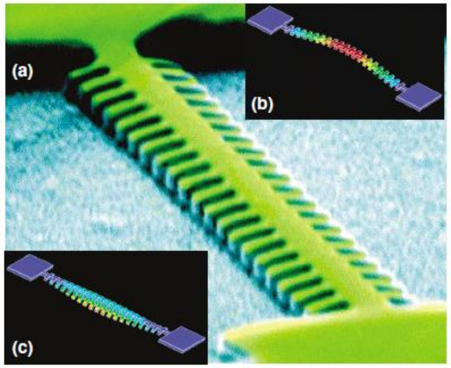
Nintendo Wii (2006)

iPhone gen 1 (2007)



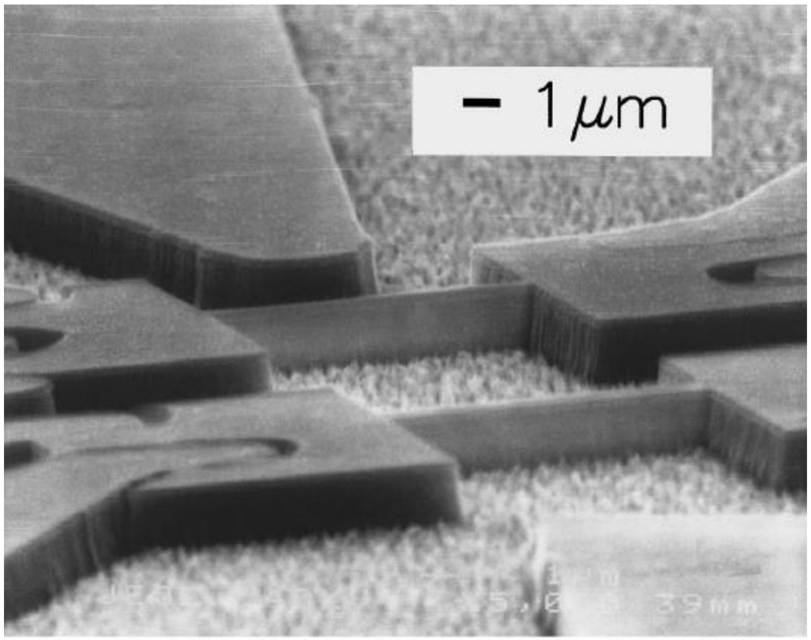
- Science:
  - Casimir effect
  - ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale

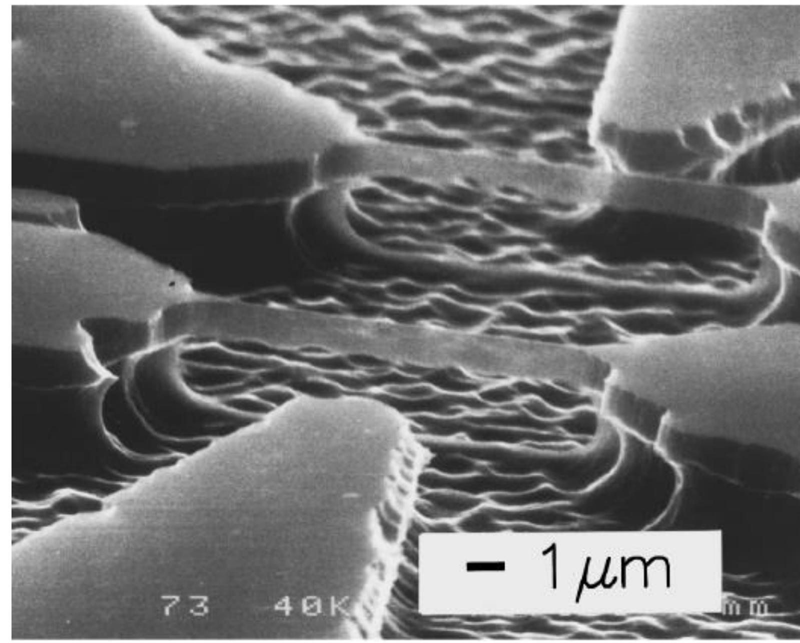




RF resonator

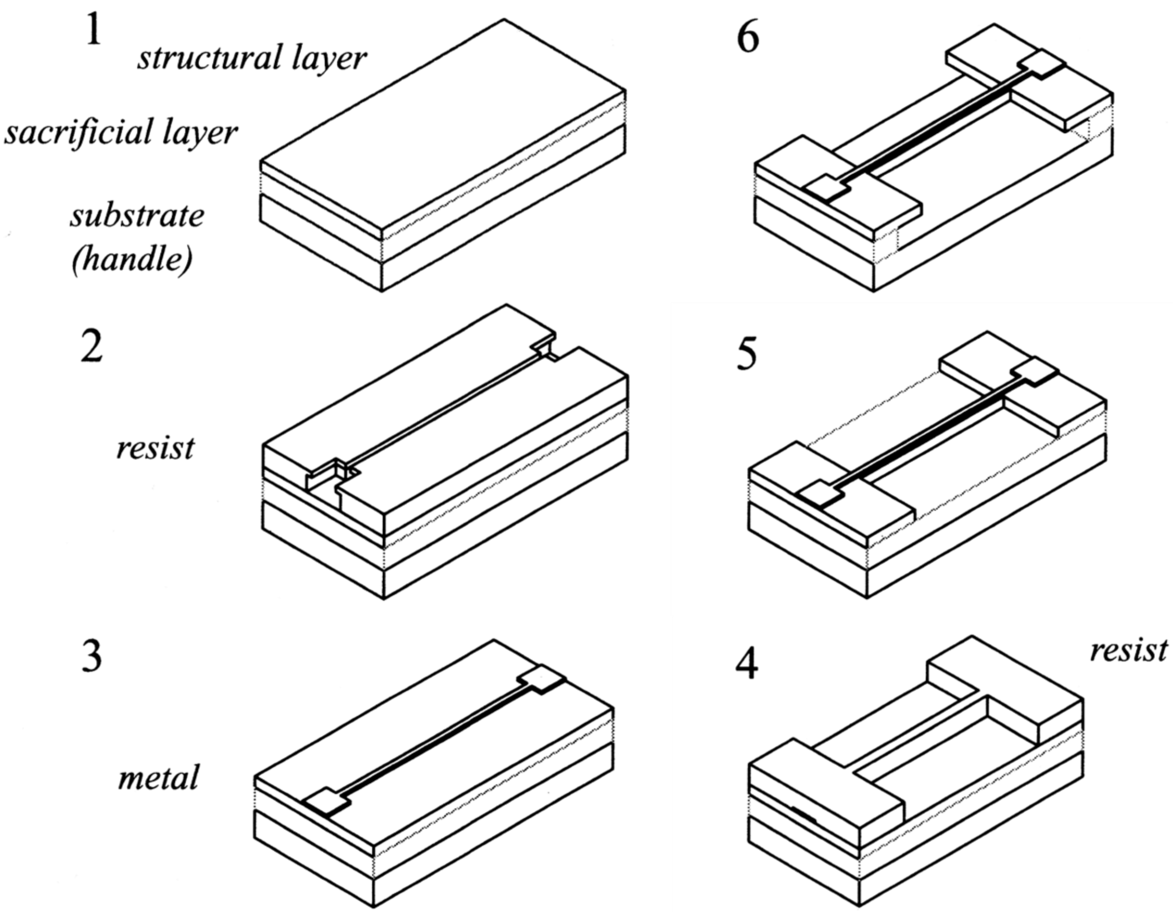
Si beams with lengths of cca 7μm, height h=0.8 μm and width w=0.33 μm





Cleland et al., APL 69, 2653 (1996)

Fabrication sequence



1: Bare substrate

2: Photoresist coating and patterning

3: Metal deposition

4: Resist coating and patterning (for etch)

5: Etch of the structural layer, resist removal

6: Etch of the sacrificial layer

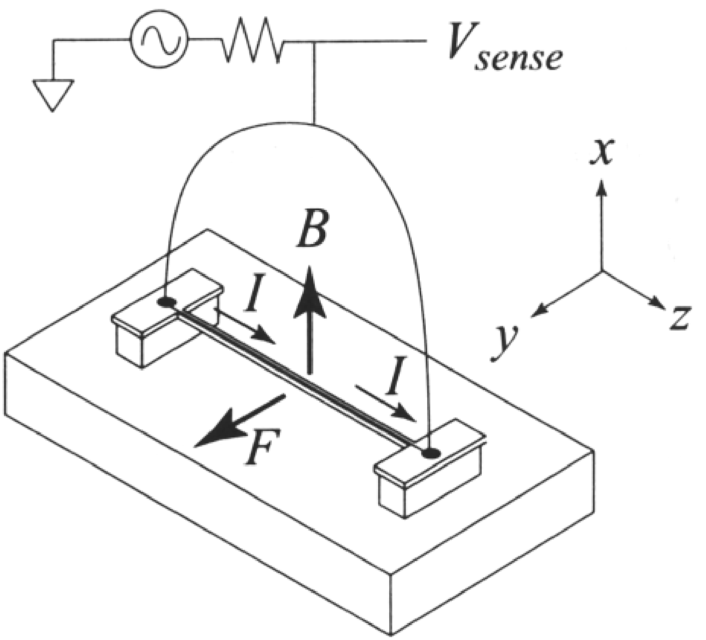
RF resonator – motion detection

Magnetomotive technique is the earliest method used for motion excitation and detection

- Device carrying rf current is placed in a magnetic field
- The magnetic field induces rf Lorentz force

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

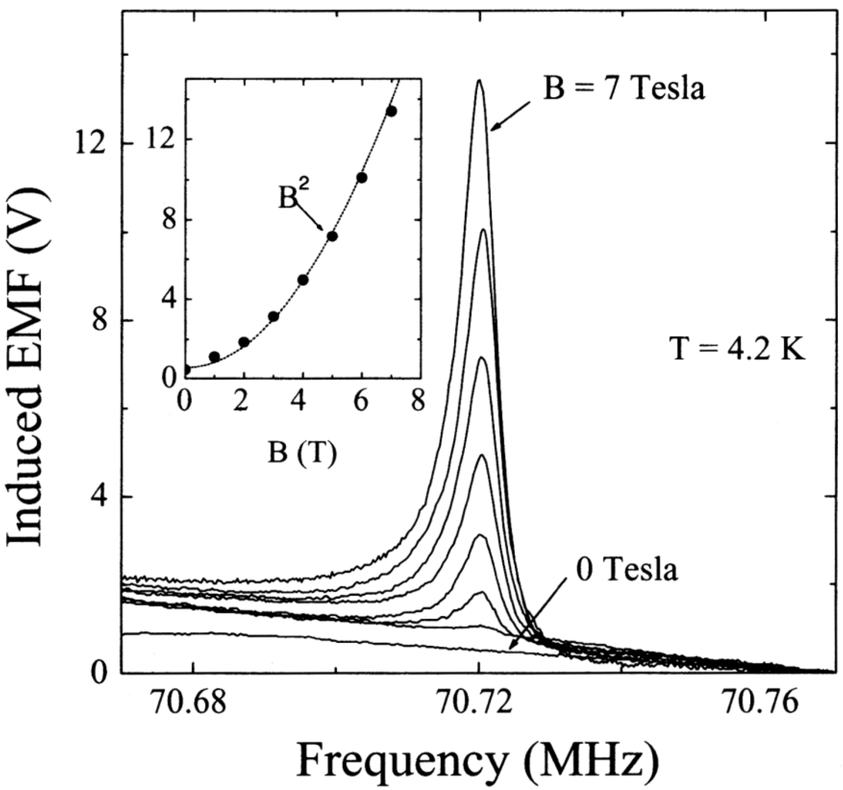
- Resulting displacement of the beam generates an electromotive force (voltage)
- This voltage is sensed, allowing the displacement to be measured
- Displacements are usually on the order of resonator widths

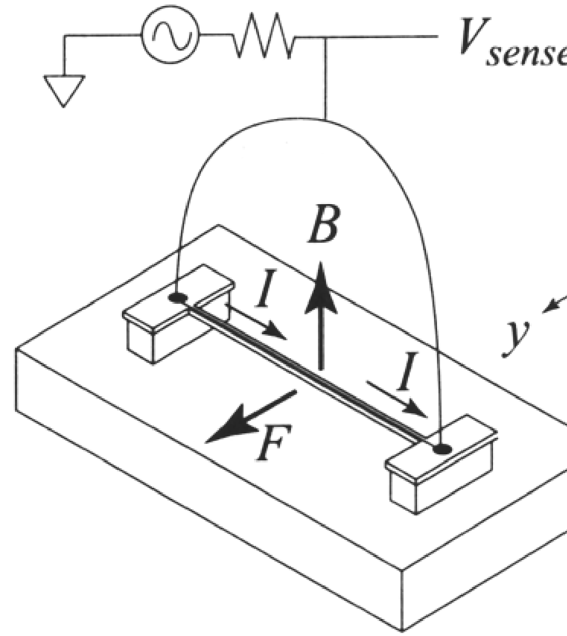


Cleland et al., APL 69, 2653 (1996)

RF resonator – motion detection

Magnetomotive technique

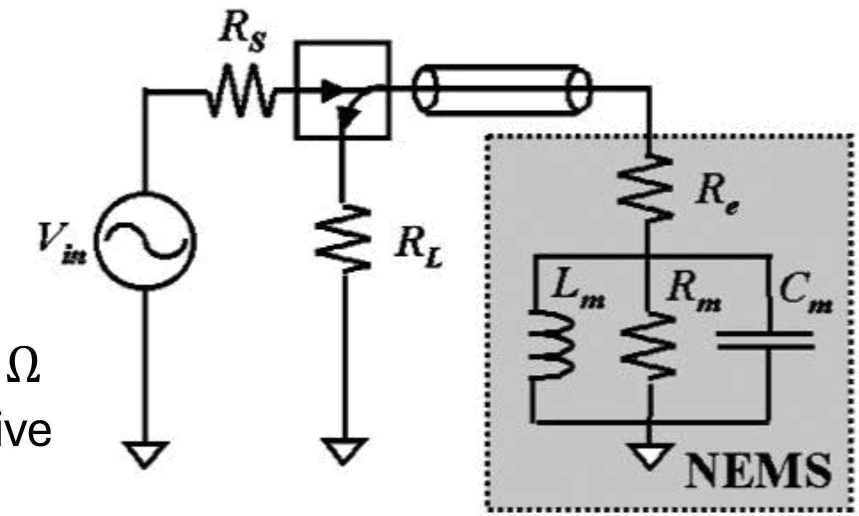




Cleland et al., APL 69, 2653 (1996)

RF resonator – motion detection

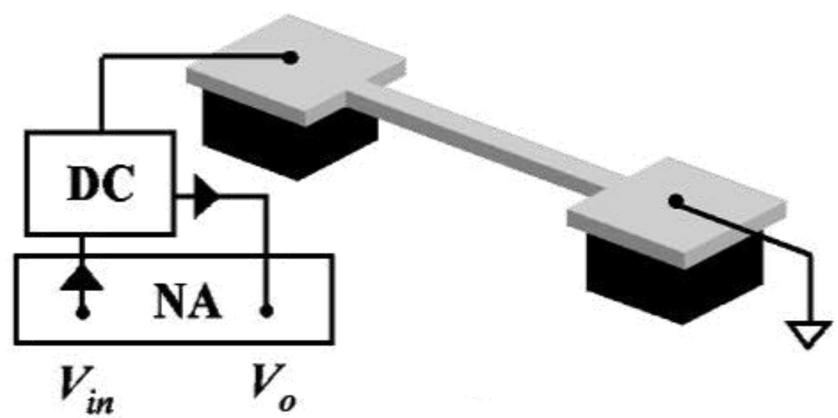
- From the electrical point of view, NEMS are modelled as a parallel RLC network with total impedance $Z_m(\omega)$ composed of resistance R_m, capacitance C_m and inductance L_m



- The NEMS is coupled to the outside world through a resistance R_e. This represents the resistance of the leads

- For NEMS made of doped Si, $R_m < 100\ \Omega$ while R_e is in the low $\mathrm{k}\Omega$ range. Capacitive and inductive losses can be neglected in first approximation:

$$Z_m \approx R_m$$



- NEMS is driven by a voltage source with an internal resistance R_s and the voltage is measured on the load R_L.

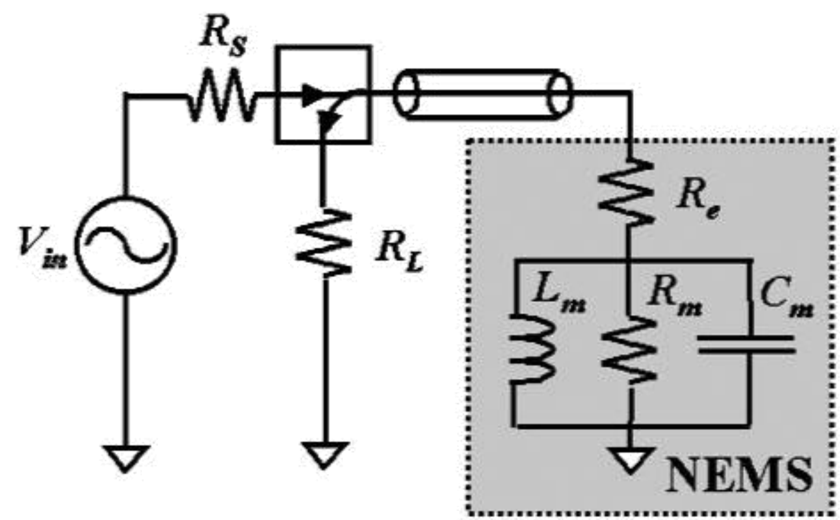
$$R_L = R_S = 50\ \Omega$$

Ekinci et al., APL 81, 2253 (2002)

RF resonator – motion detection

In this case, the voltage on the load can be approximated as:

$$V_0(\omega) \approx V_{in}(\omega)\frac{R_e + Z_m(\omega)}{R_L + [R_e + Z_m(\omega)]}$$



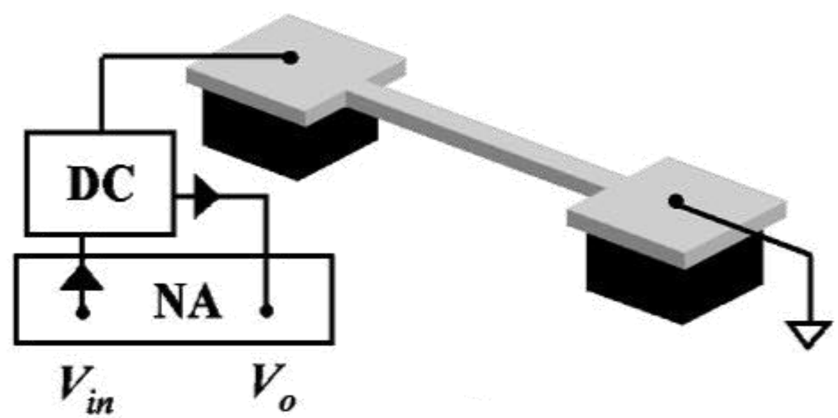
With the following numbers:

$$R_e = 2\ k\Omega, R_m = 100\ \Omega$$

$$Z_m = 100\ \Omega, R_L = 50\ k\Omega$$

we get

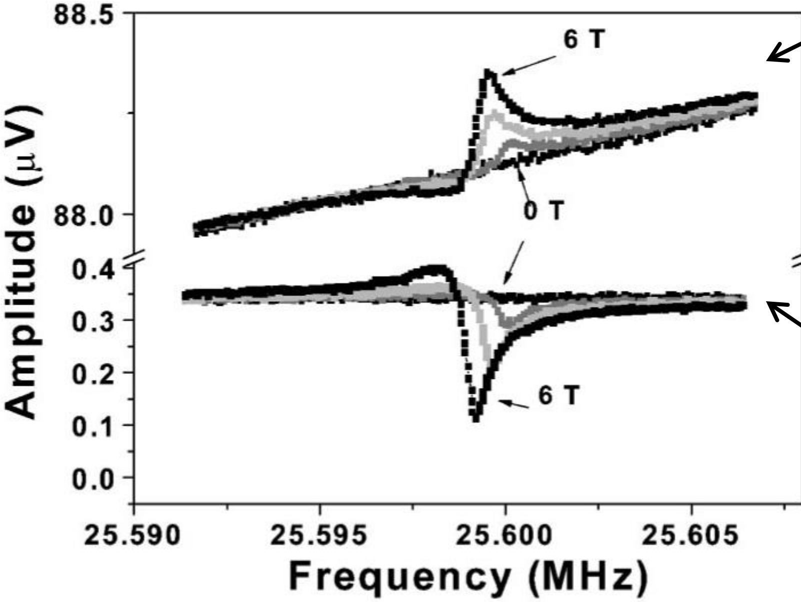
$$V_0(\omega) \approx V_{in}(\omega) \times 0.98$$



Ekinci et al., APL 81, 2253 (2002)

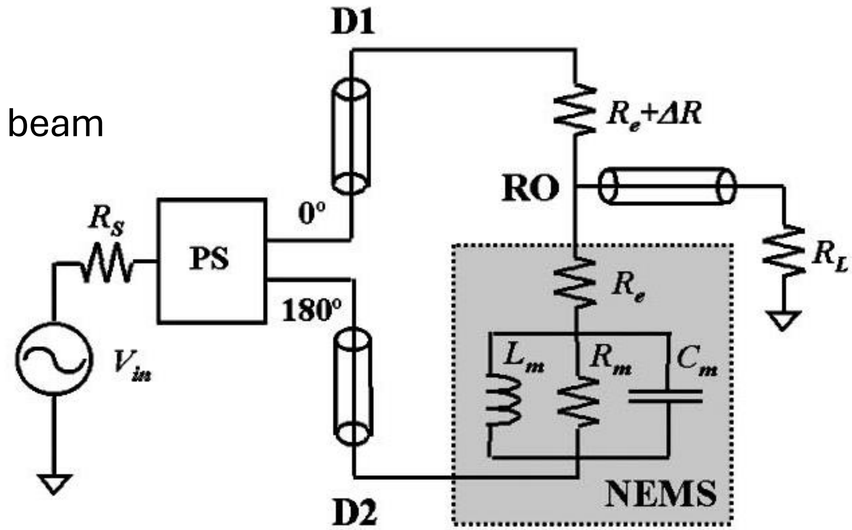
RF resonator – motion detection

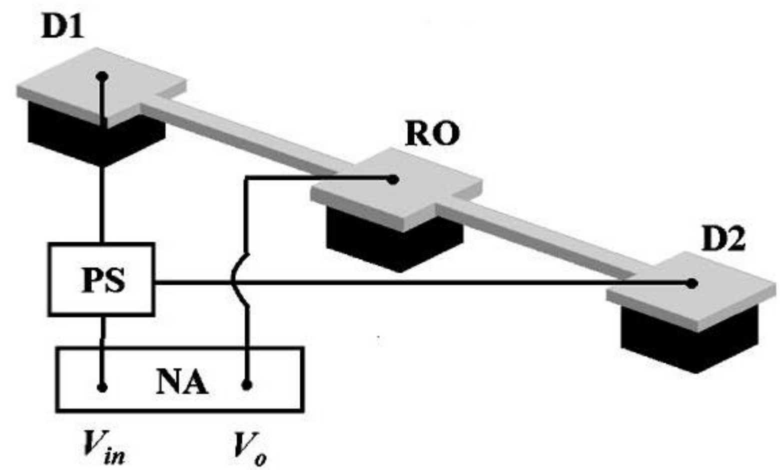
Sensitivity can be improved by using detection schemes that involve bridges



single beam

bridge





Ekinci et al., APL 81, 2253 (2002)

Sensitive mass detection

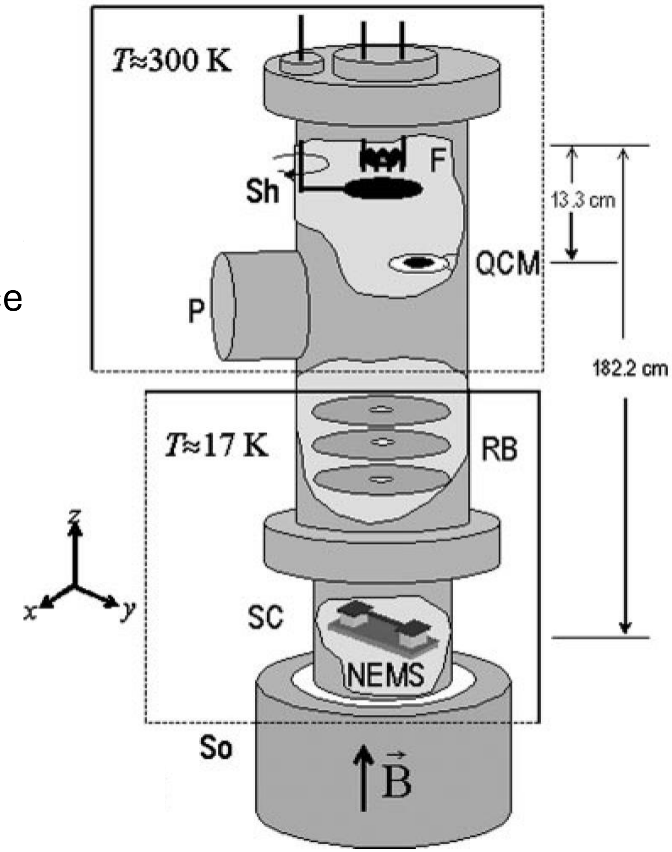
SiC NEMS

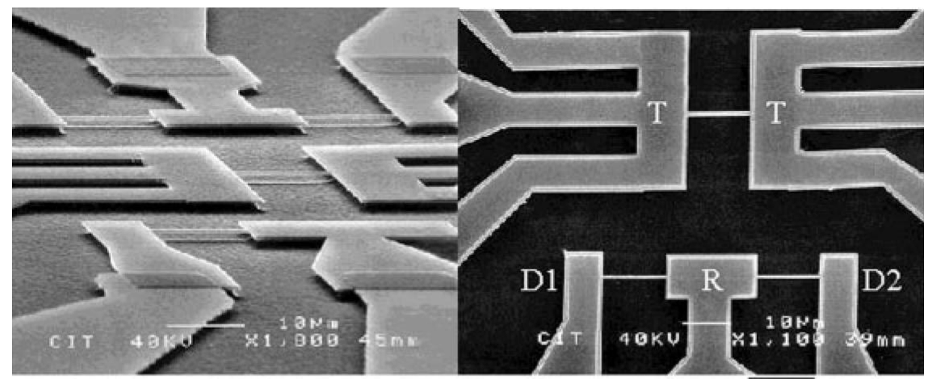
filament and shutter

quartz microbalance

apertures

NEMS





Ekinci et al., APL 84, 4469 (2004)

Sensitive mass detection

- Resonant frequency:

$$\omega^2 = \frac{k}{m}$$

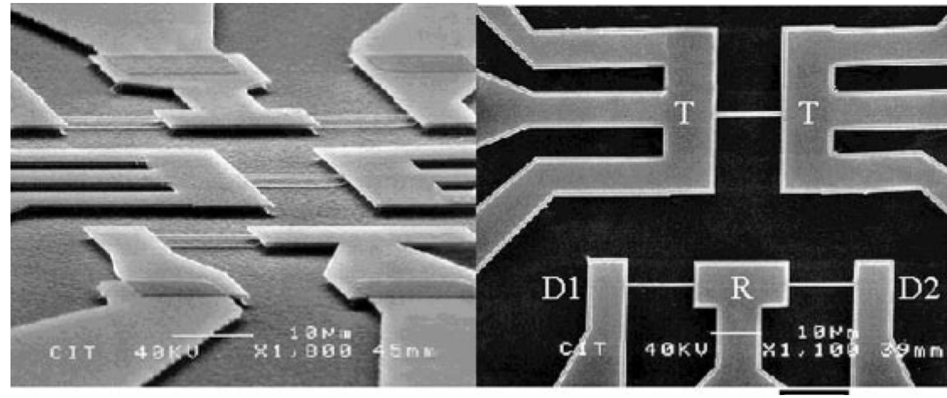
- Mass detection:

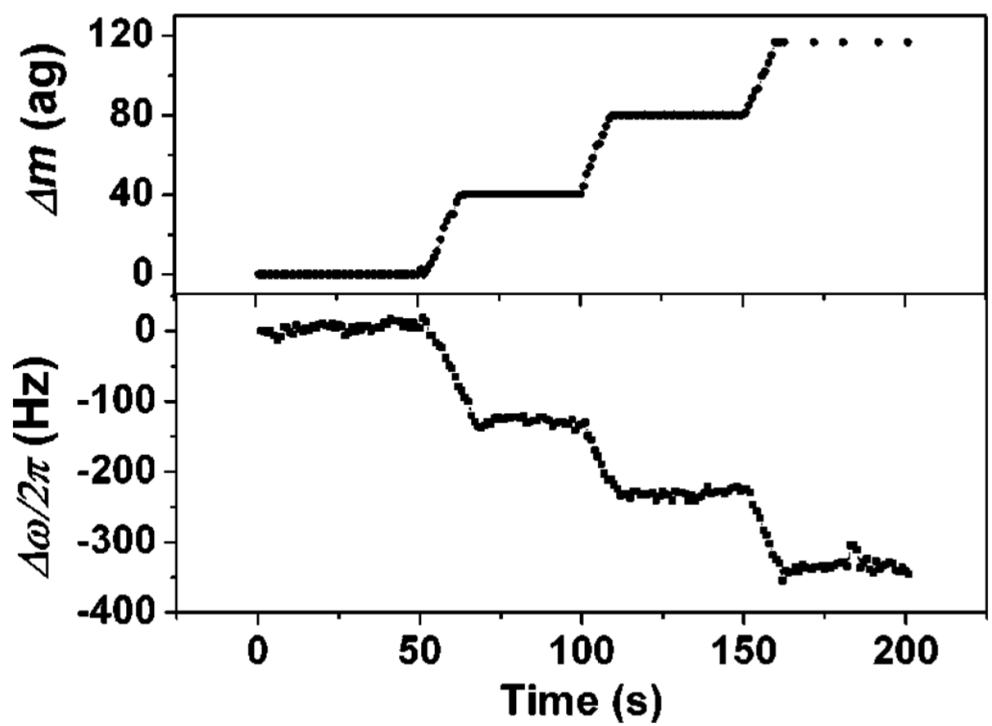
$$2\omega d\omega = -\frac{k}{m^2}dm = -\omega^2\frac{dm}{m}$$

$$\left|\frac{dm}{m}\right| = 2\frac{d\omega}{\omega}$$

- Resolution: 2.5 ag = 2.5×10^{-18} g

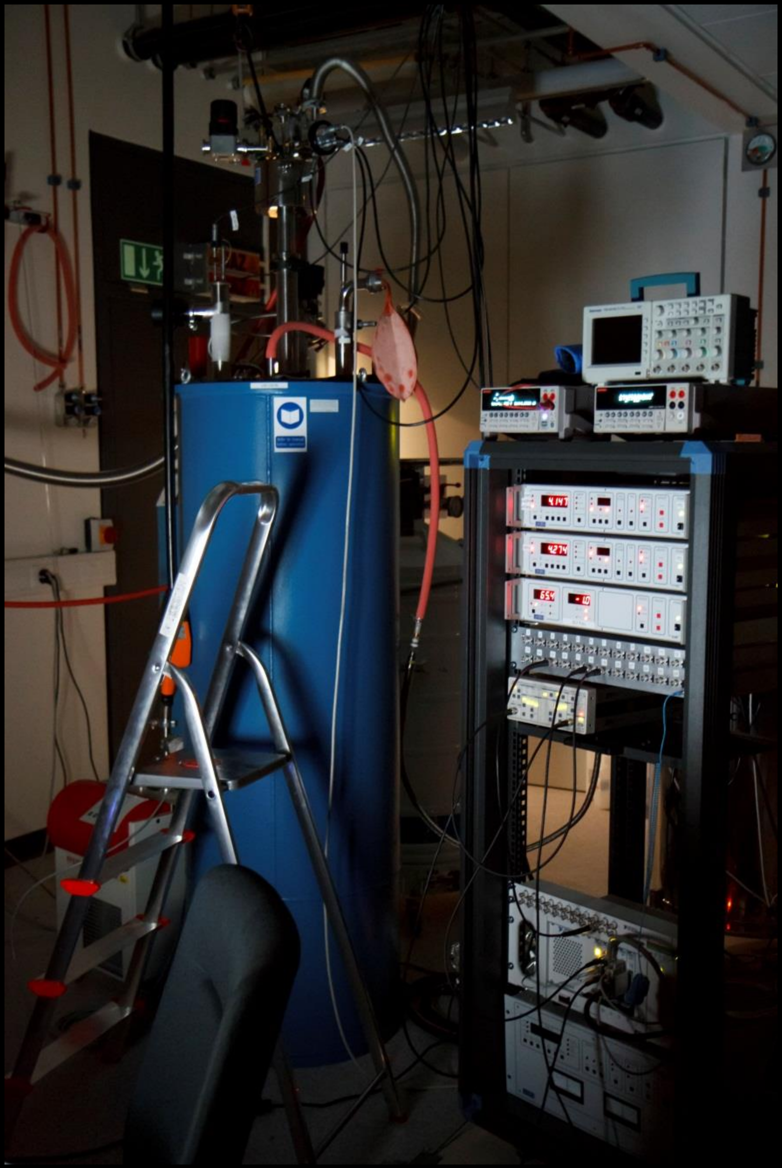
  cca 7500 Au atoms





Ekinci et al., APL 84, 4469 (2004)

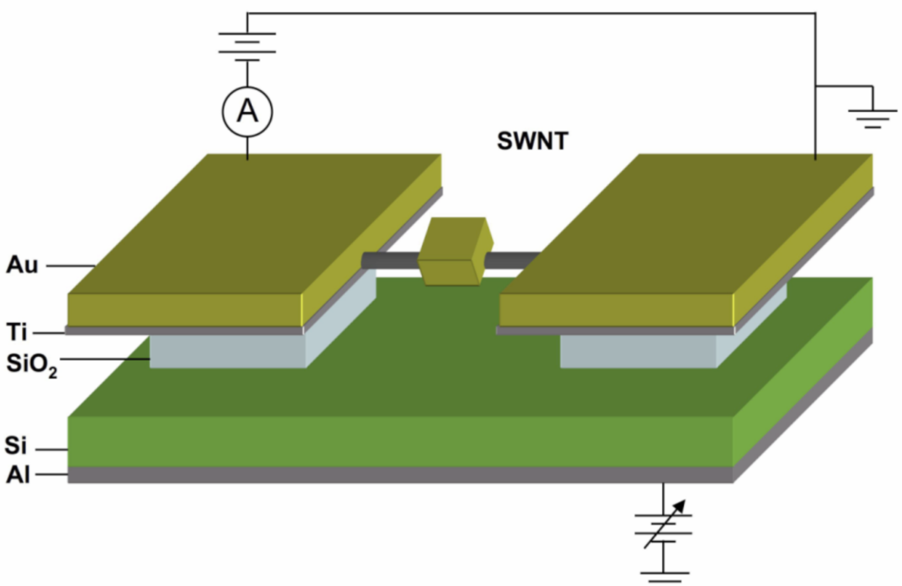
This is what you need to get 6 Tesla

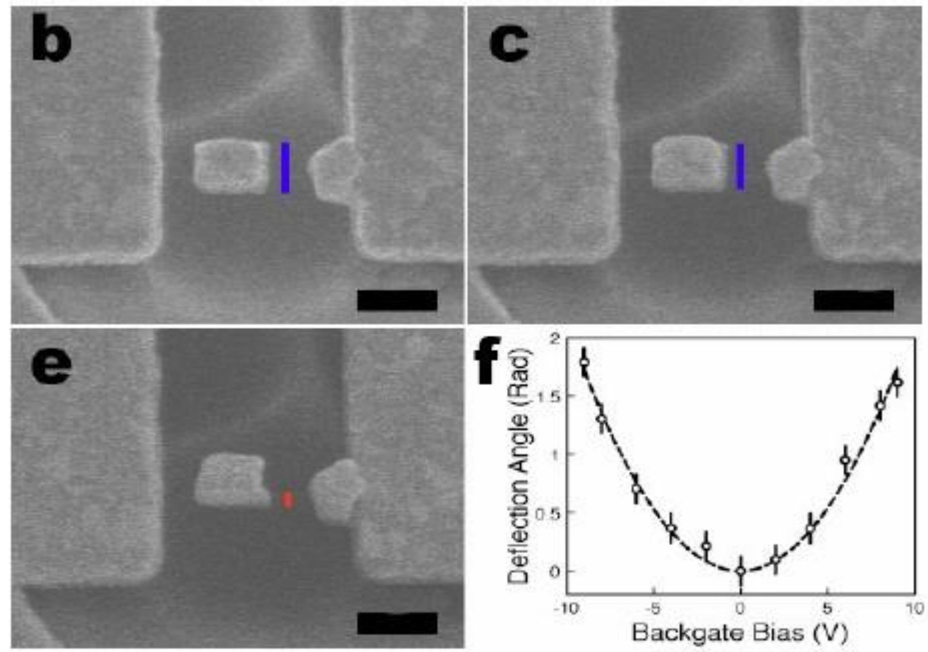


Superconducting magnet in the Kis group

Torsional oscillators with CNTs

- Idea – look for a physical property that changes with deformation
- Carbon nanotube – based torsional oscillator



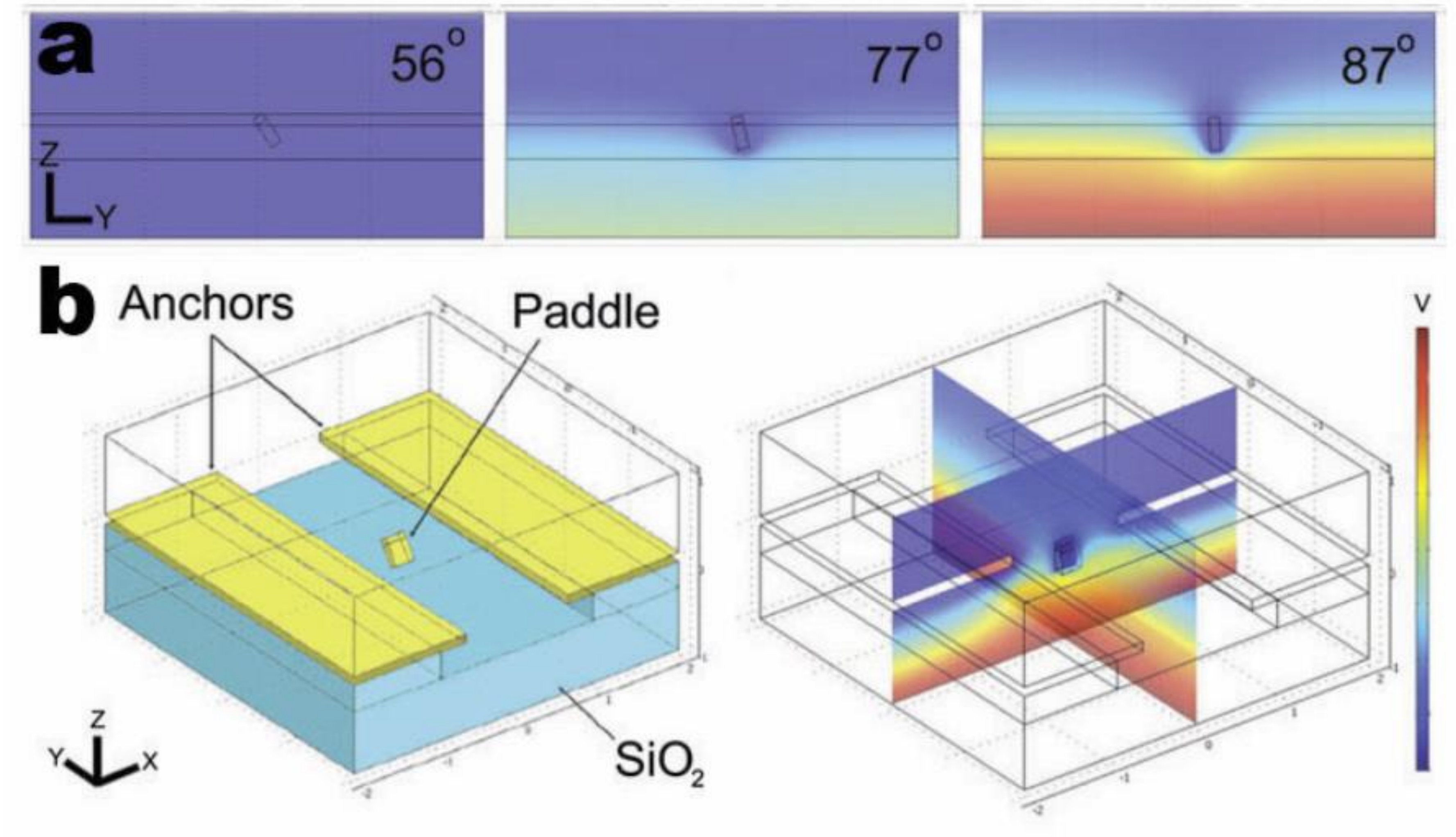


A. Hall, PhD thesis, Chappel Hill 2007

Papadakis et al., PRL 93, 146101 (2004)

Torsional oscillators with CNTs

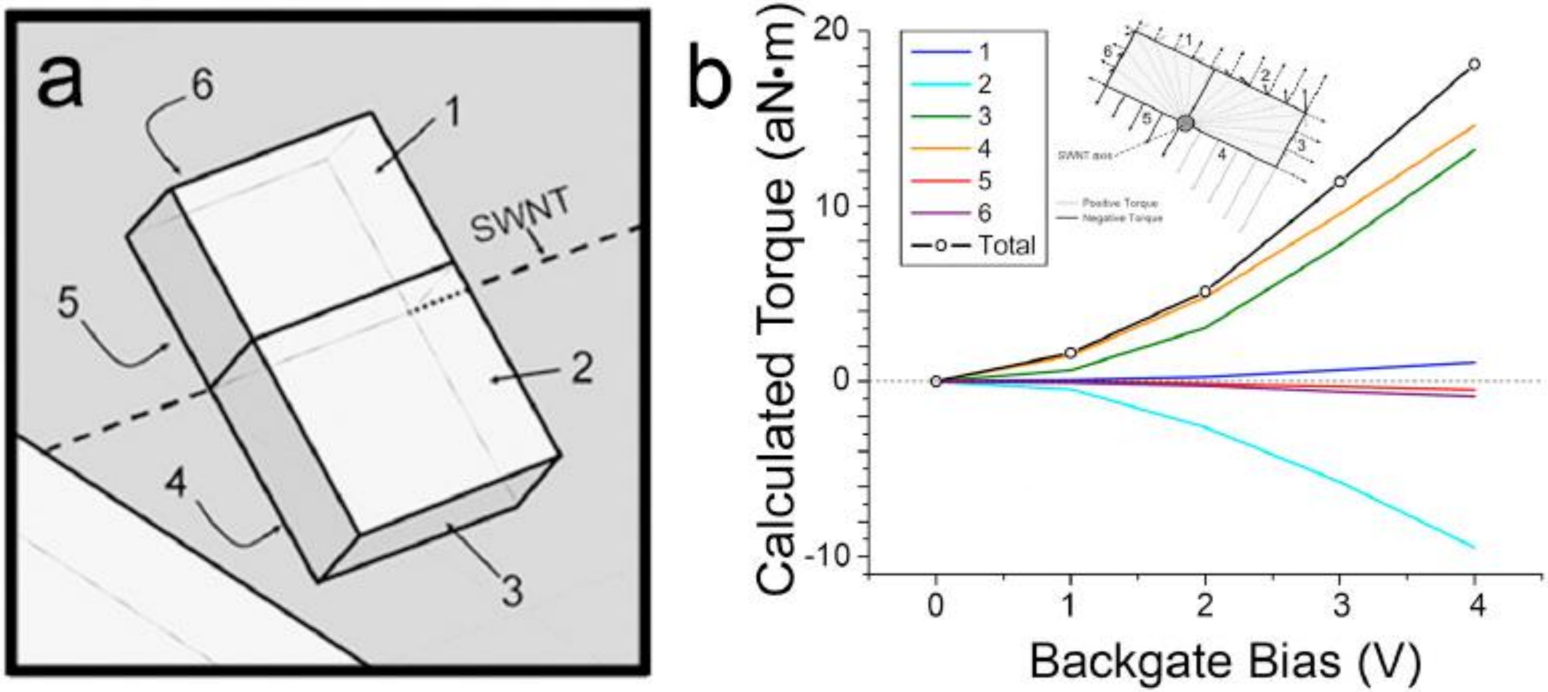
- Carbon nanotube – based torsional oscillator



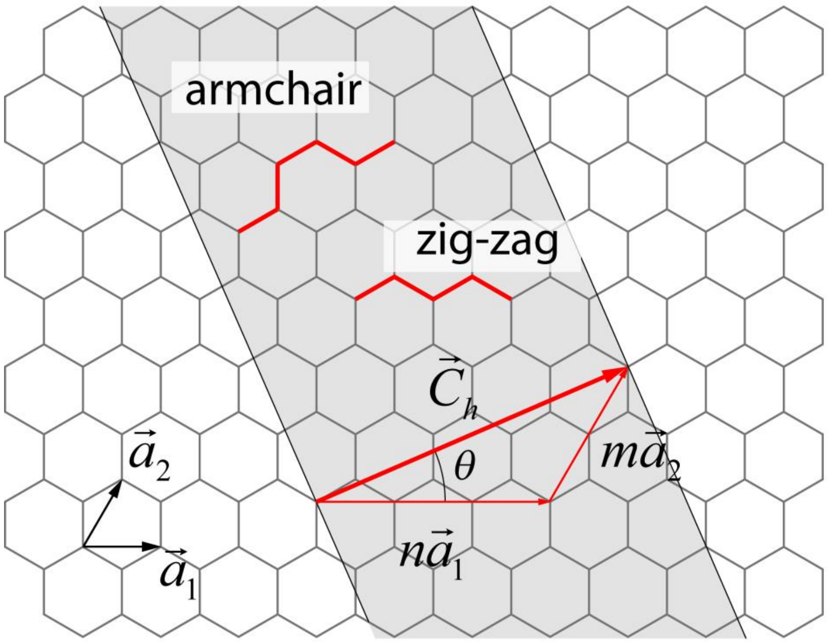
A. Hall, PhD thesis, Chappel Hill 2007

Papadakis et al., PRL 93, 146101 (2004)

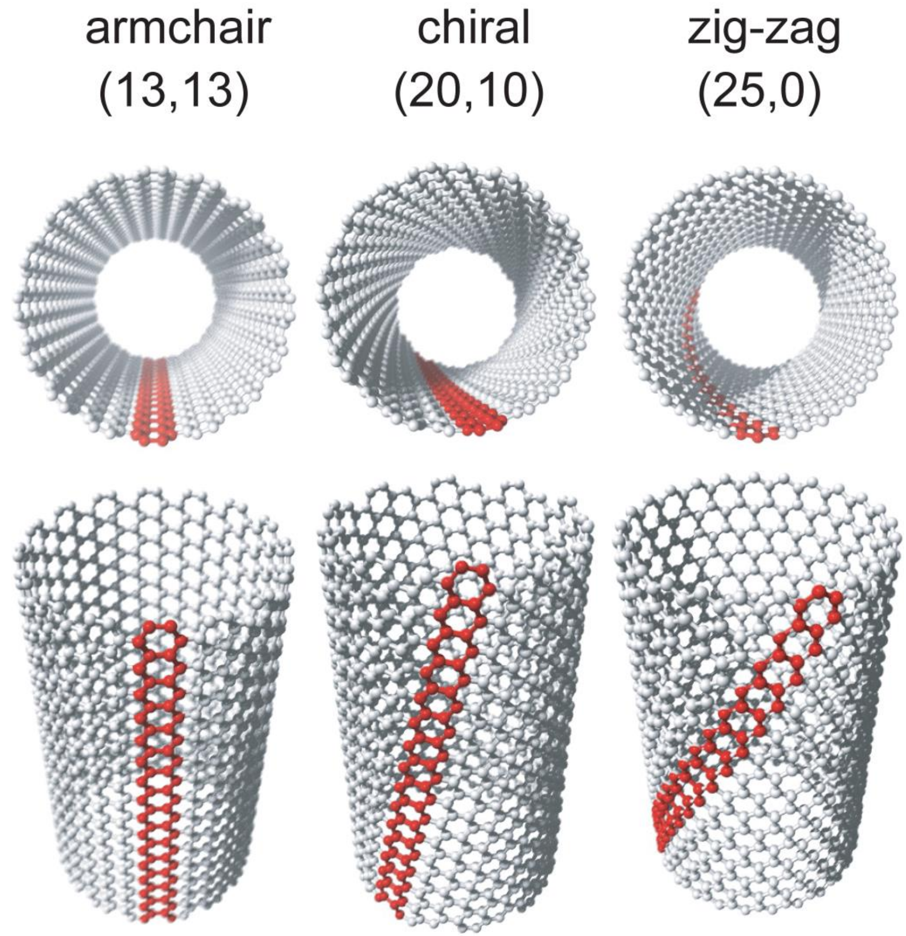
Torsional oscillators with CNTs



Nanotube chirality

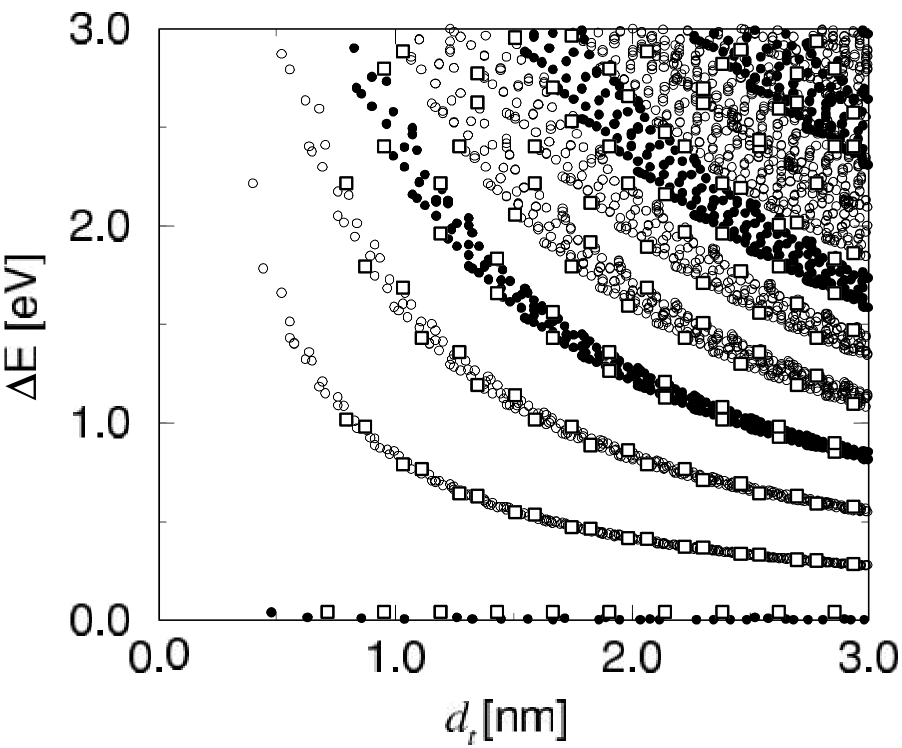


$$\vec{C}_h = n\vec{a}_1 + m\vec{a}_2 = (n,m)$$

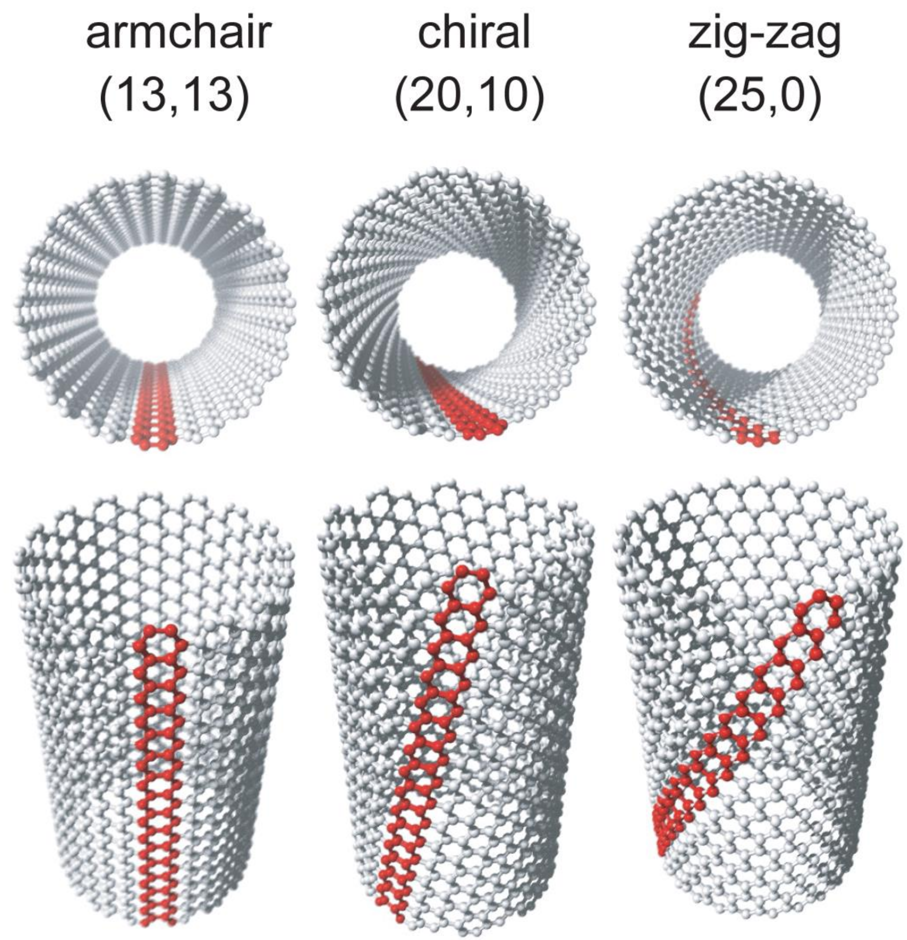


Nanotube chirality

Kataura plot

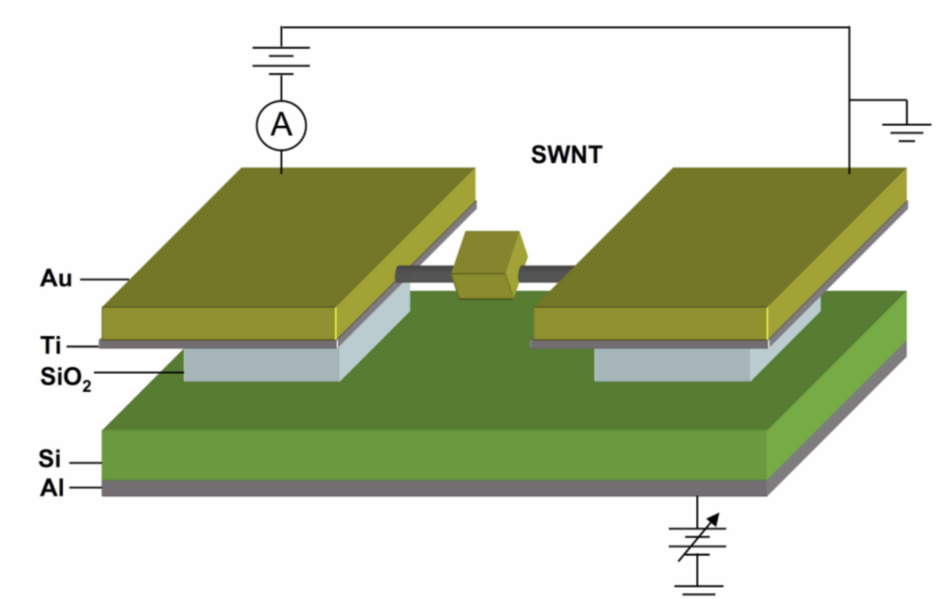


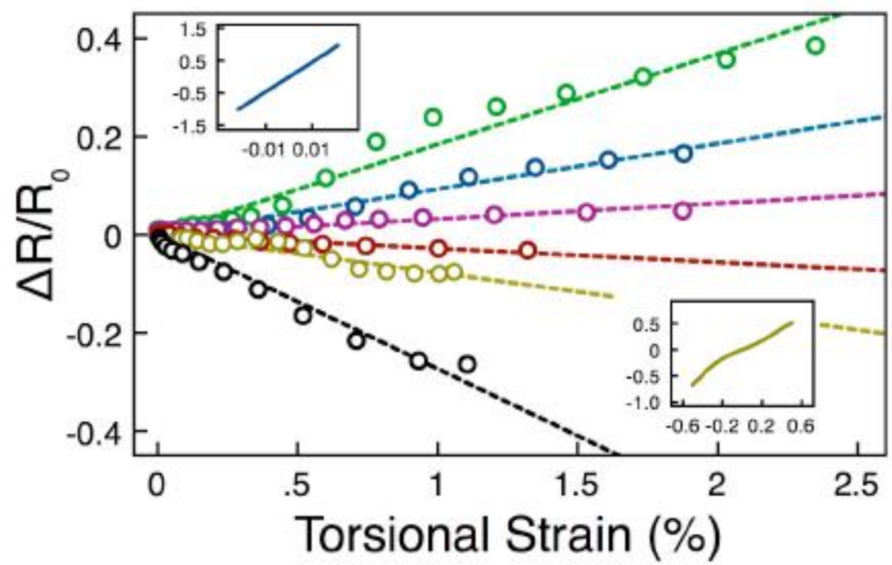
Kataura et al, Synthetic Metals (1999)



Torsional oscillators with CNTs

Carbon nanotube – based torsional oscillator

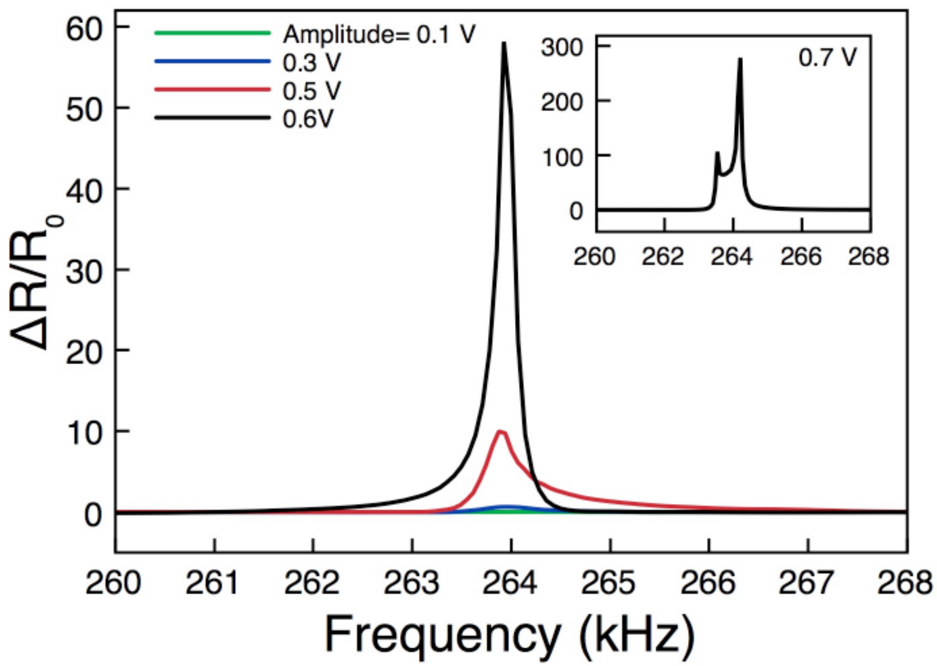


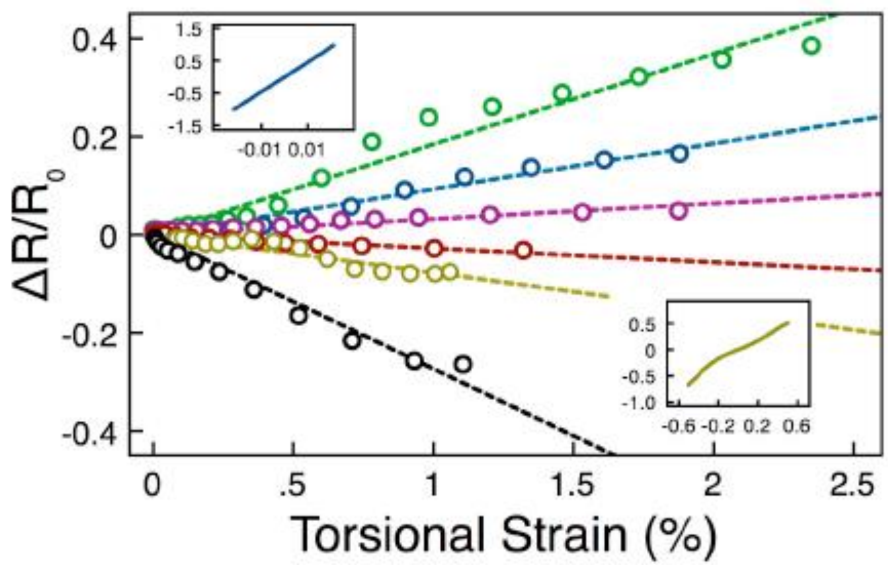


Hall et al., Nat Nano 2, 413 (2007)

Torsional oscillators with CNTs

Carbon nanotube – based torsional oscillator



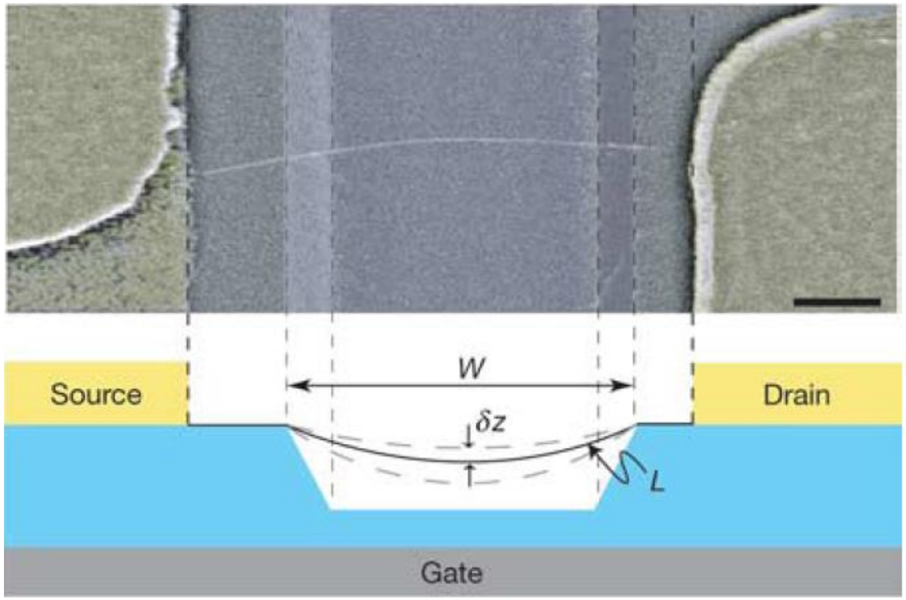


Hall et al., Nat Nano 2, 413 (2007)

Electrostatic actuation

- NEMS based on a carbon nanotube
- CNT is suspended above a back-gate and is actuated electrostatically
- The energy stored in the capacitor with CNT as one and back gate as the other electrode is:

$$E = \frac{C_g {V_g}^2}{2}$$



- The electrostatic force is the gradient of the energy:

$$F_{el} = -\frac{dE}{dz} = -\frac{1}{2}\left[\frac{dC_g}{dz}\right]{V_g}^2 = -\frac{1}{2}\left[\frac{dC_g}{dz}\right]\left[{V_g}^{DC} + {v_g}^{AC}\right]^2 \approx -\frac{1}{2}\left[\frac{dC_g}{dz}\right]{V_g}^{DC}\left[{V_g}^{DC} + 2{v_g}^{AC}\right]$$

- where the gate voltage has both a static DC component and a small AC component ($V_g = {V_g}^{DC} + {v_g}^{AC}; {v_g}^{AC} = {V_g}^{AC}\cos(\omega t)$) so that $\left(v_g^{AC}\right)^2$ is negligibly small

Sazonova et al., Nature 431, 284 (2004)

Motion detection

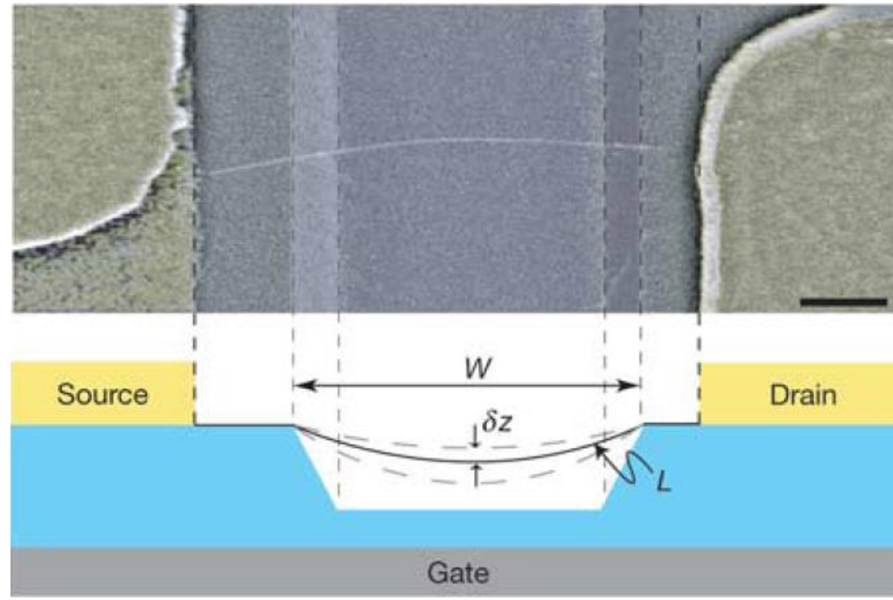
- The oscillating beam is a so-called small bandgap semiconducting carbon nanotube ($E_g = 10\ meV$)

- The conductance change of such semiconducting nanotubes is proportional to the charge induced on the tube:

$$\delta q = \delta(C_g V_g) = \underbrace{C_g \delta V_g}_{\text{"normal" gating}} + \underbrace{V_g \delta C_g}_{\neq 0 \text{ for a moving nanotube}}$$

"normal" gating

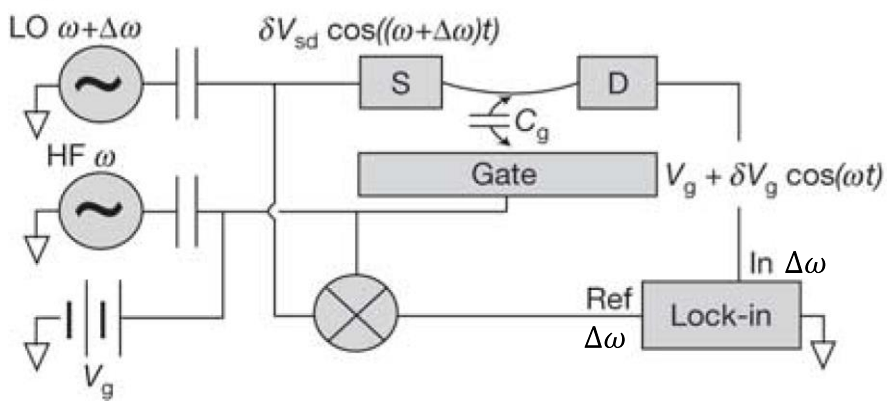
$\neq 0$ for a moving nanotube

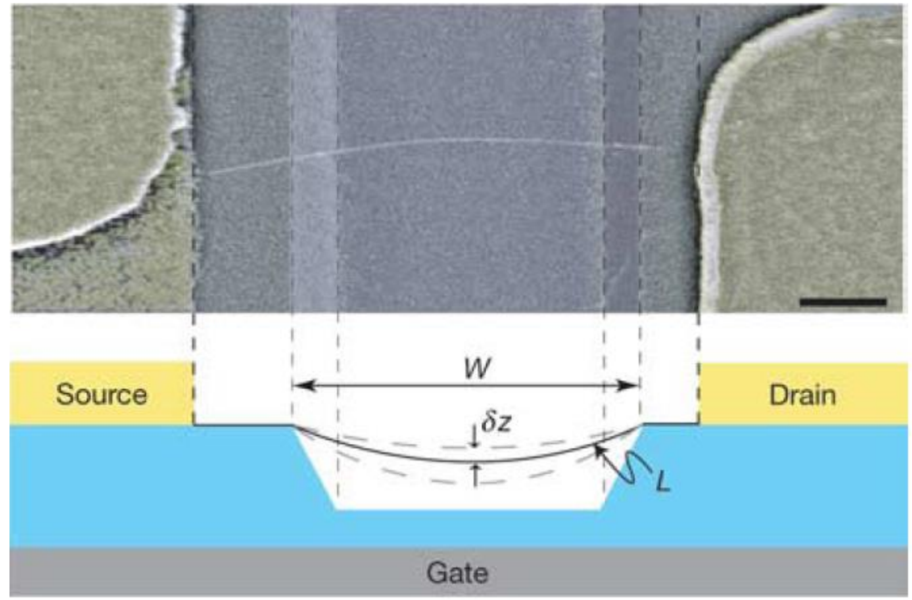


Sazonova et al., Nature 431, 284 (2004)

Motion detection

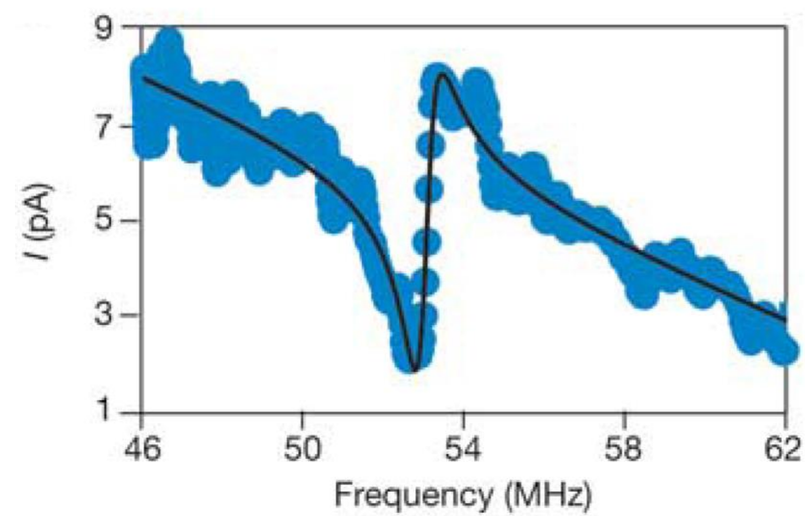
- Mixing technique, allowing use of low-bandwidth lock-in amplifiers





$$\delta I^{lock-in} = \delta G \delta V_{sd} =$$
$$= \frac{1}{2\sqrt{2}} \frac{dG}{dV_g} \left[\delta V_g + \underbrace{V_g^{DC} \frac{\delta C_g}{C_g}} \right] \delta V_{sd}$$

sensitive to motion

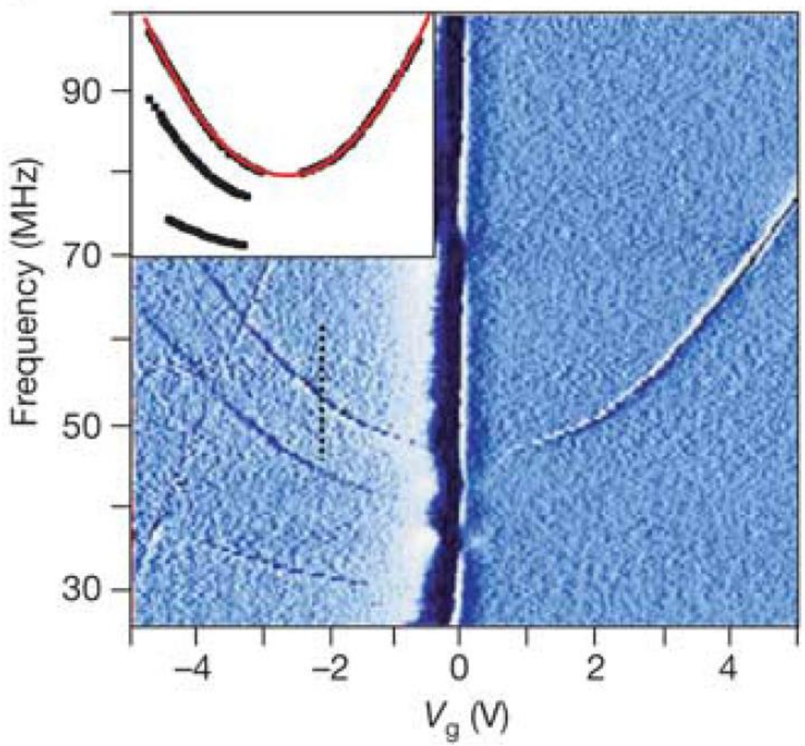


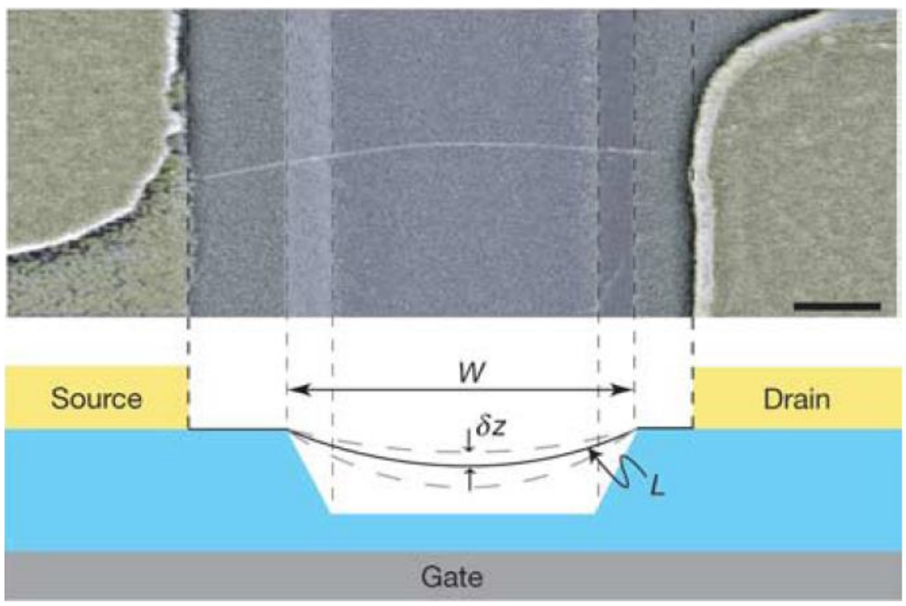
Sazonova et al., Nature 431, 284 (2004)

Sazonova, PhD thesis (2004), page 62

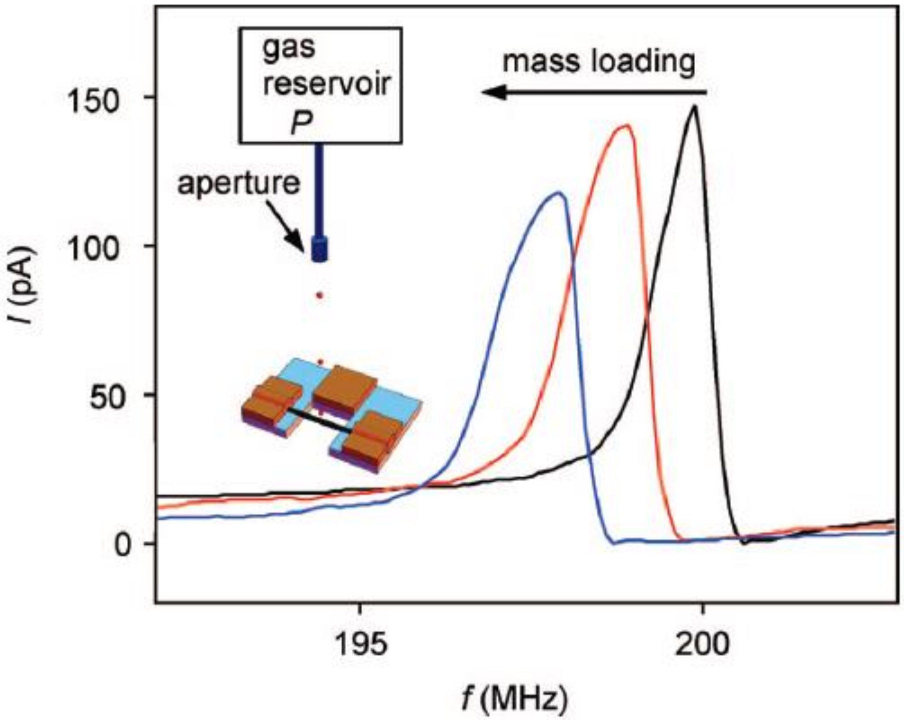
Tunability

- Static voltage on the gate can induce tension in the suspended nanotube
- Changing the tension in the nanotube results in a shifting resonance frequency

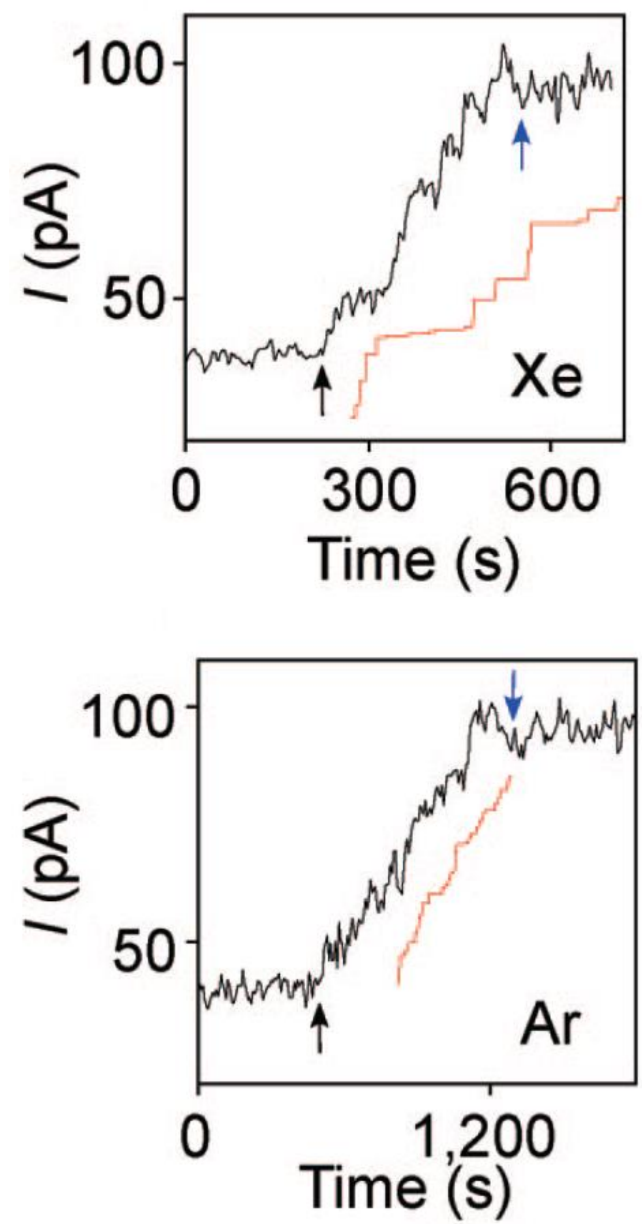




Sensitive mass detection using nanotubes



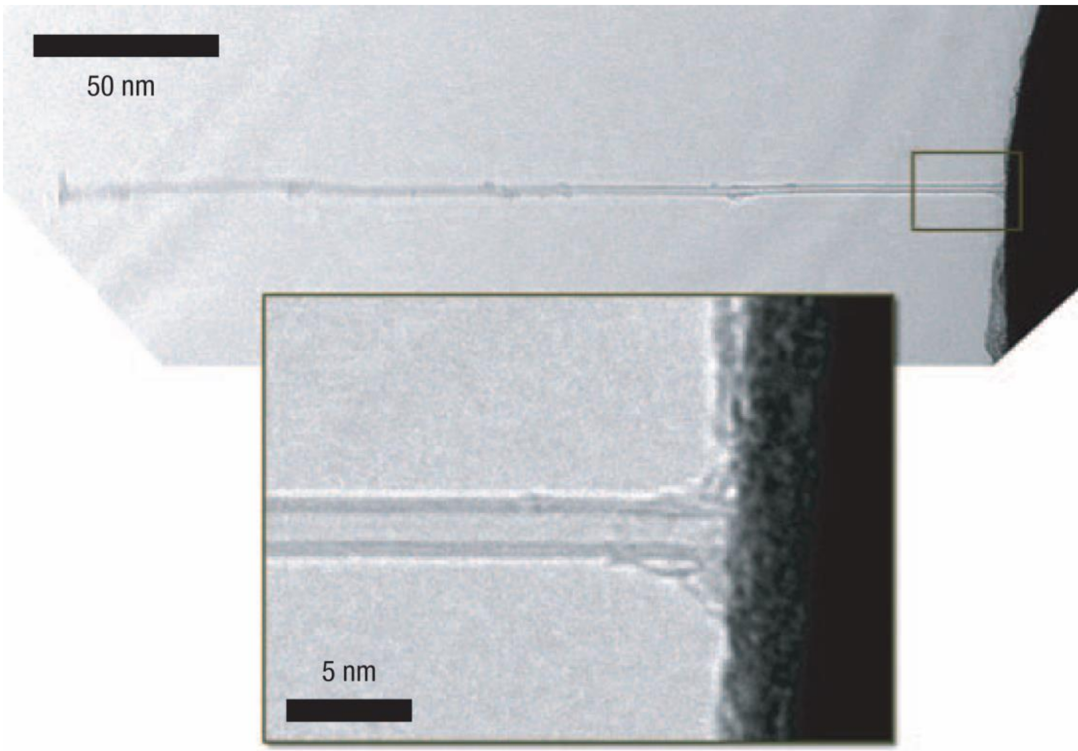
$$\frac{\Delta f_0}{f_0} \approx -\frac{m}{m_0} \sin(\pi a/L) \sin[\pi(L-a)/L]$$

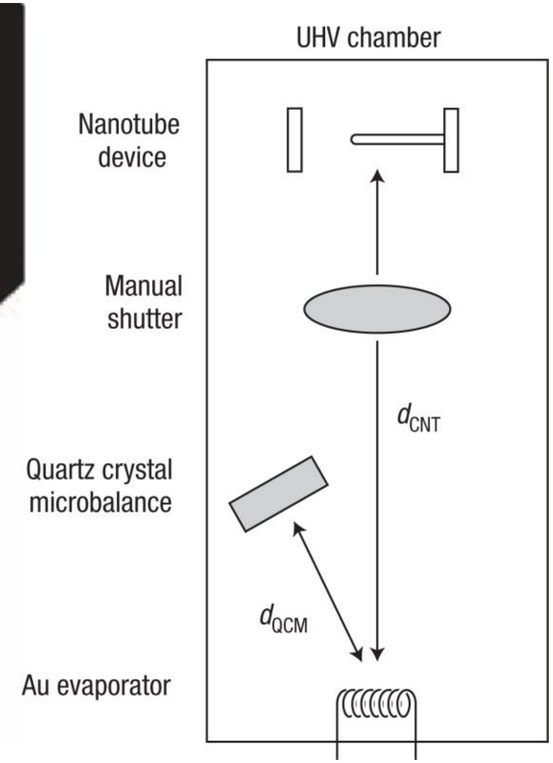


Chiu et al. Nano Lett. 8 4342 (2008)

Sensitive mass detection using CNTs

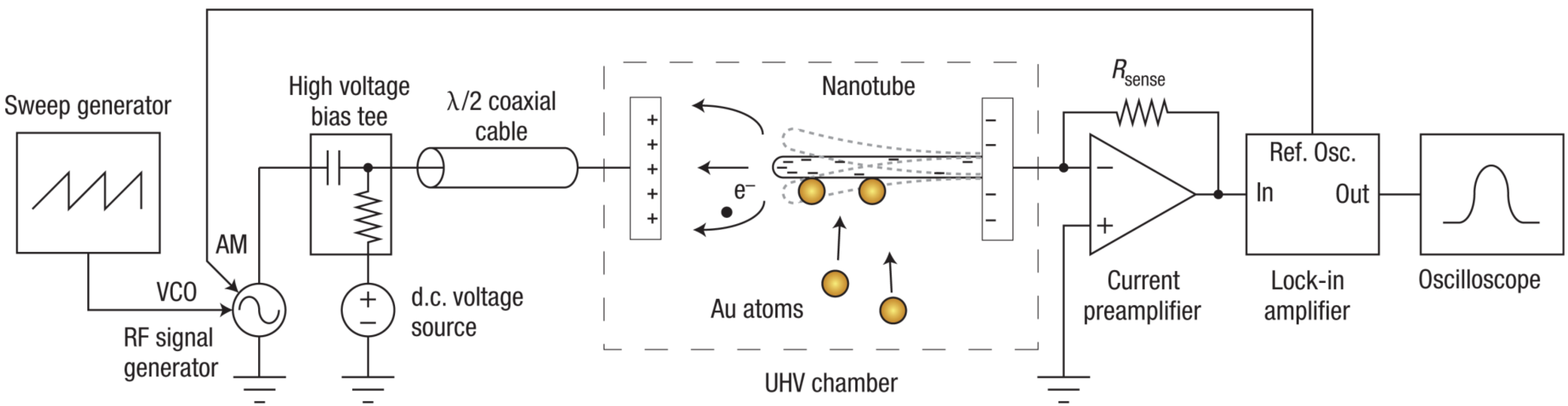
- Works at room temperature





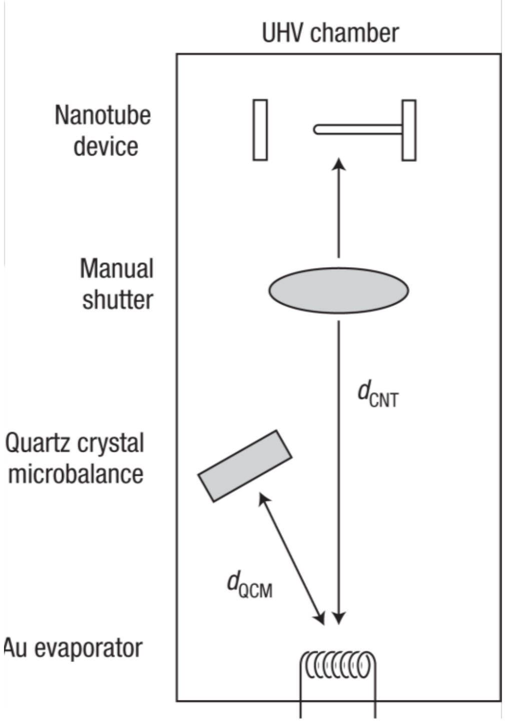
Jensen et al., Nature Nanotech. 3, 533 (2008)

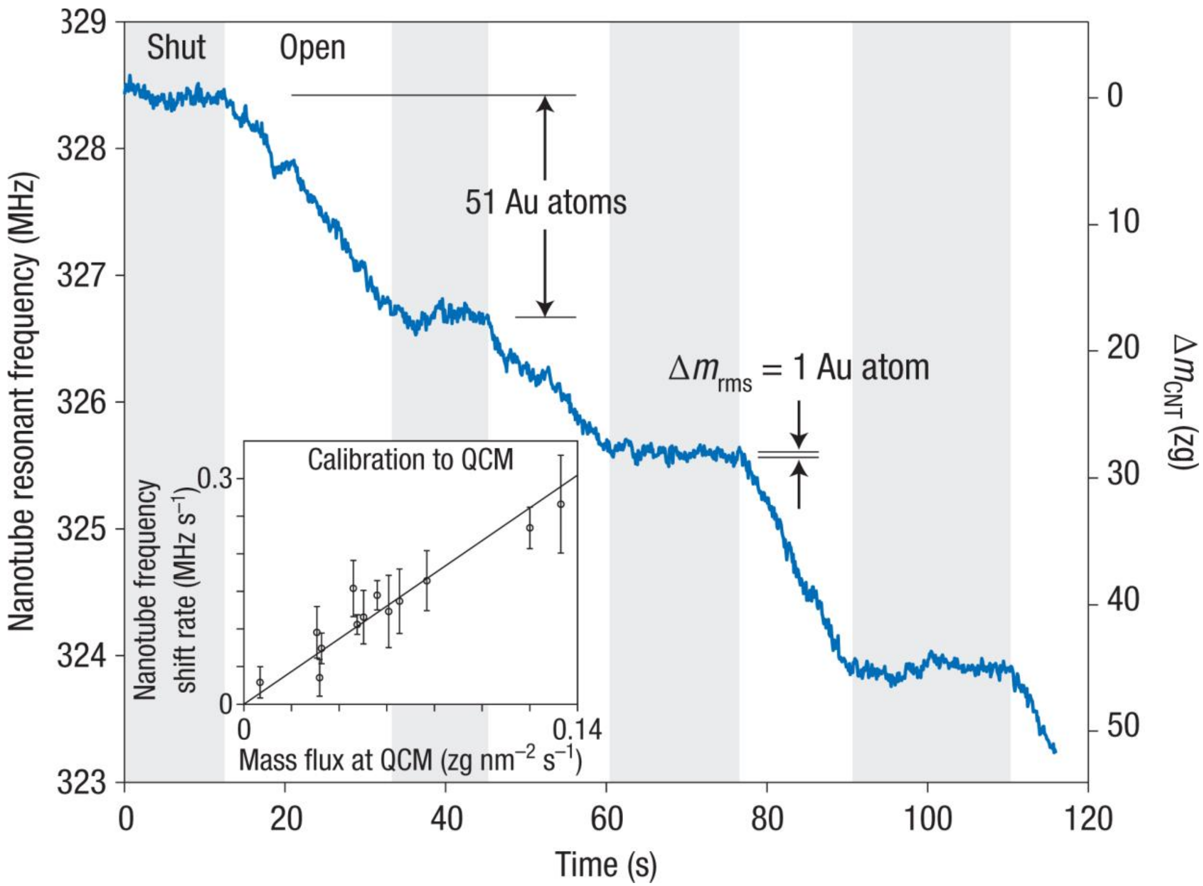
Sensitive mass detection using CNTs



Jensen et al., Nature Nanotech. 3, 533 (2008)

Sensitive mass detection using CNTs

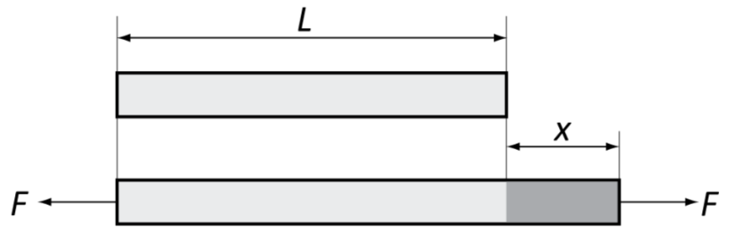




Theoretical background: mechanics

Basic definitions (stress and strain)

- Let us consider a bar like the one on figure



- When we subject it to an axial force F, it will elongate by an amount x

- If the force acts at the center of the cross-section, the uniform **stress** σ will be given by:

$$\sigma = \frac{F}{A}$$ where A is the beam's cross-sectional area

- If the bar is made of a homogeneous material, the axial **strain** ε will be:

$$\varepsilon = \frac{x}{L}$$

- Hooke's law states that stress σ is proportional to strain ε (for small deformations):

$$\sigma = E\varepsilon$$

or in a more familiar form

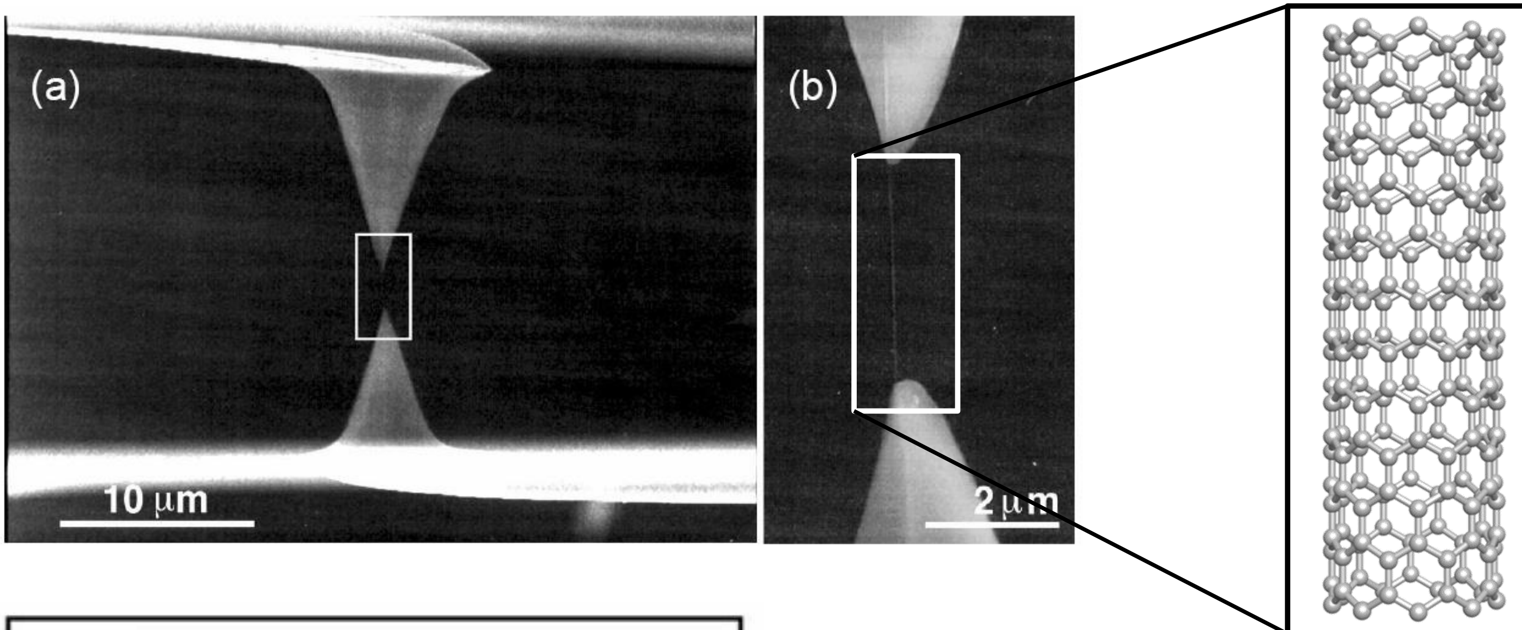
$$\frac{F}{A} = E\frac{x}{L} \quad \rightarrow \quad F = E\frac{A}{L} \cdot x$$

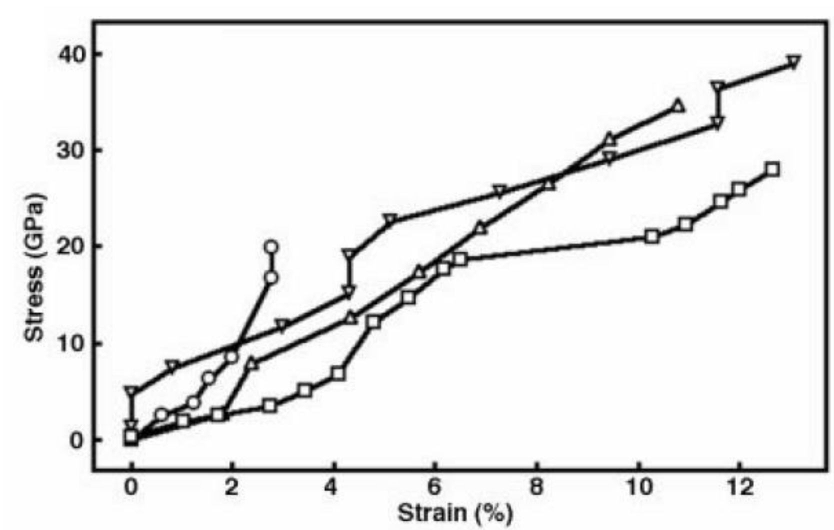
which can be written as

$$F = kx$$ (Hooke's law)

Hooke's law at the nanoscale

- Still holds – example of carbon nanotubes

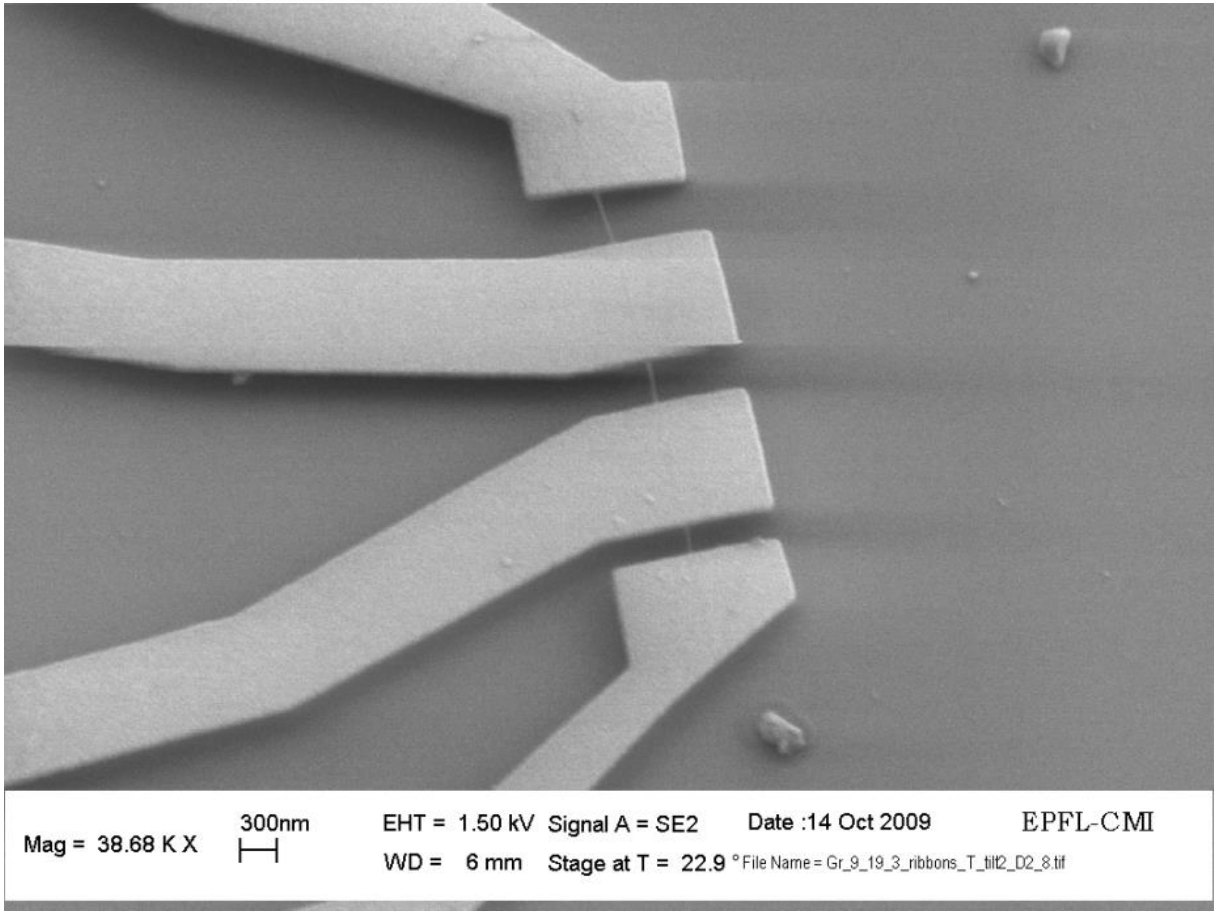




$$E_{\text{Young}} = \frac{1}{V_0}\frac{\partial^2 E}{\partial \varepsilon^2}$$

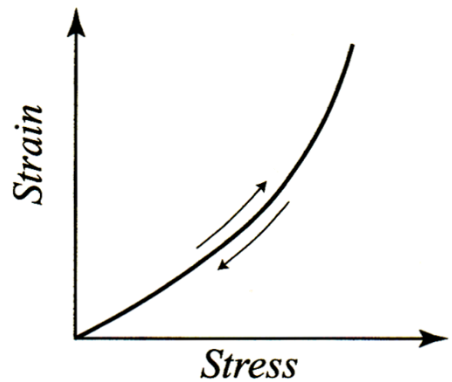
Yu et al., Science 287, 637 (2000)

2D nanoribons @ LANES

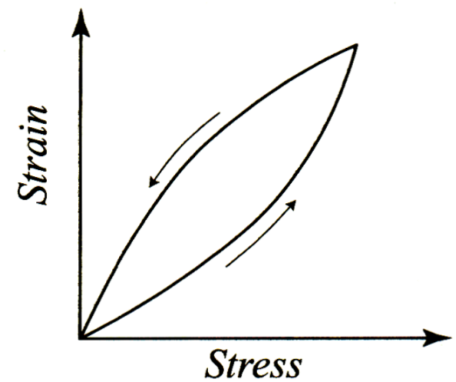


Types of mechanical response

- Materials can be classified according to the way they respond to stress as:
  - **elastic** – strain is uniquely determined by the stress
  - **anelastic** – stress has multiple values and hysteretic behavior



Stress-strain curve for an elastic material

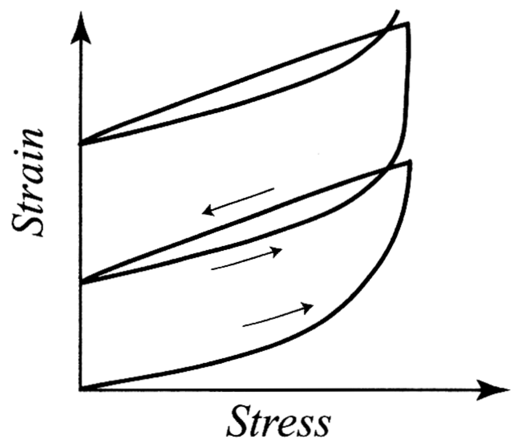


anelastic material

- Typical examples of anelastic materials: glass, rubber
- Their mechanical response is also time-dependent – the size of the hysteresis loop depends on the rate of strain
- Elastic materials fully recover their original shape and size when the strain is released
- **Better choice for NEMS/MEMS: elastic**

Types of mechanical response

- **Elastic materials** are further classified as **elastic linear** and **elastic nonlinear**, depending on whether the stress and strain are proportional to one another
- **Another classification: ductile vs. brittle**
- **Ductile materials** are elastic for small strains but can be deformed permanently with sufficiently high strain – typical examples are metals



Stress-strain curve for a ductile material

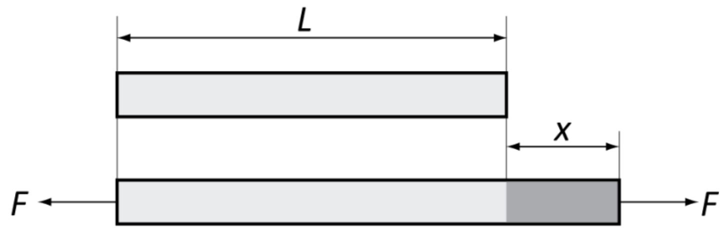
- **Brittle materials** have a linear elastic response up to the breaking point

  Examples: usually covalently bonded materials such as Si, Ge, diamond

- **Better choice for MEMS/NEMS: brittle**

Basic definitions (stress and strain)

- Let us consider a bar like the one on figure



- When we subject it to an axial force *F*, it will elongate by an amount *x*

- If the force acts at the center of the cross-section, the uniform **stress** σ will be given by:

$$\sigma = \frac{F}{A}$$ where *A* is the beam's cross-sectional area

- If the bar is made of a homogeneous material, the axial **strain** will be:

$$\varepsilon = \frac{x}{L}$$

- Hooke's law states that stress is proportional to strain (for small deformations):

$$\sigma = E\varepsilon$$

or in a more familiar form

$$\frac{F}{A} = E\frac{x}{L} \quad \rightarrow \quad F = E\frac{A}{L} \cdot x$$

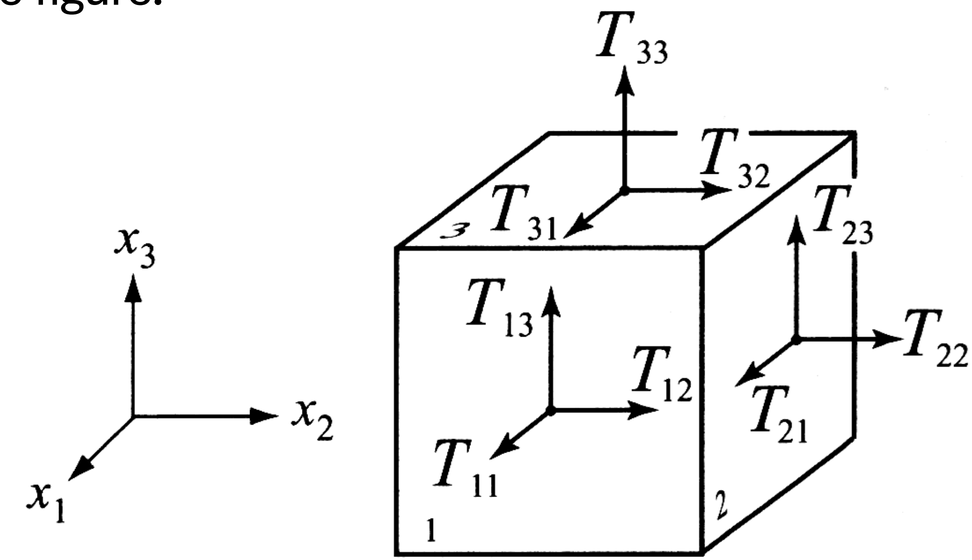
which can be written as

$$F = kx$$ (Hooke's law)

Linear elastic response (most general description)

- Let us consider a volume element like the one shown on the figure, subjected to stress $\mathbf{T}$ with components shown on the figure.

- Surface forces are represented by the product of the stress components and the areas on which they act



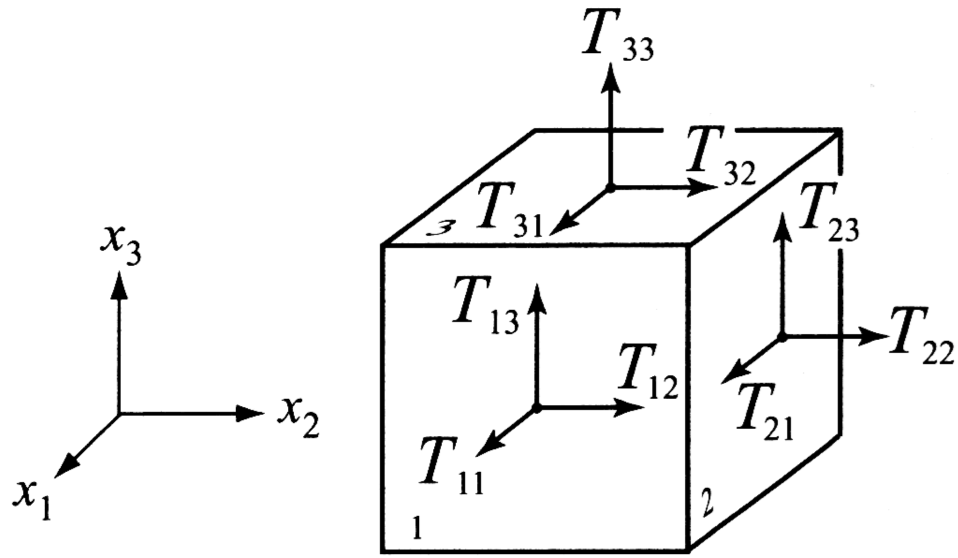
- The 9 stress components can be written down as a tensor:

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

Linear elastic response

- If we require that the body is in static equilibrium, the stress tensor becomes symmetric and can be simplified to:

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12} & T_{22} & T_{23} \\ T_{13} & T_{23} & T_{33} \end{bmatrix}$$



- Associated with the stress tensor is the corresponding strain tensor:

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{where} \quad S_{ij} = \frac{\partial u_i}{\partial x_j}$$

and u is the displacement field

($u(x)$ tells how much the point at coordinate x moved from the starting point)

Linear elastic response

- The most general linear relation relating stress **T** to strain **S** is given by:

$$T_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} \alpha_{ijkl} S_{kl}$$ where constants α_{ijkl} are the **elastic moduli**

- This is the generalisation of the expression: $\sigma = \varepsilon E$

- In principle, the values of the elastic moduli can vary from point to point within a solid; we however allways assume that the material is **homogeneous**, so the elastic moduli are independent of the coordinates

- In other words, we assume that the entire object is made of the same material

- There are in principle $3^4 = 81$ distinct values of $\alpha_{ijkl} S_{kl}$, but as the stress and the strain tensors $\mathbf{T}$ and $\mathbf{S}$ are both symmetric, this reduces the number of independent values to 36

- Further reductions can be made depending on the crystalline structure of the material, all the way to only 2 elastic constants for the case of **isotropic** materials
- Typical isotropic materials are polycristalline materials such as poly Si, poly Si_3N_4 or amorphous materials (SiO_2) – these are often used in NEMS/MEMS

Linear elastic response

- The tensors $\mathbf{T}$ and $\mathbf{S}$ are symmetric, so instead of 3x3=9 independent values we have 3+3=6 so it is easier to write the tensors as vectors with 6 components
- The correspondence between the stress tensor and vector is this one:

$$\mathbf{T} = \begin{bmatrix} \tau_1 & \tau_6 & \tau_5 \\ \tau_6 & \tau_2 & \tau_4 \\ \tau_5 & \tau_4 & \tau_3 \end{bmatrix}$$

- We can define an equivalent, simplified form for the strain tensor but with unavoidable factors of 2 in some of the terms:

$$\varepsilon_1 = S_{11} = \frac{\partial u_1}{\partial x_1}, \quad \varepsilon_2 = S_{22} = \frac{\partial u_2}{\partial x_2}, \quad \varepsilon_3 = S_{33} = \frac{\partial u_3}{\partial x_3}$$

$$\varepsilon_4 = 2S_{23} = \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}, \quad \varepsilon_5 = 2S_{13} = \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}, \quad \varepsilon_6 = 2S_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}$$

Linear elastic response

- Written out in array format the strain tensor **S** looks like this:

$$\mathbf{S} = \begin{bmatrix} \varepsilon_1 & \varepsilon_6/2 & \varepsilon_5/2 \\ \varepsilon_6/2 & \varepsilon_2 & \varepsilon_4/2 \\ \varepsilon_5/2 & \varepsilon_4/2 & \varepsilon_3 \end{bmatrix}$$

- Using these simplified definitions above, we can rewrite the elasticity relation

$$T_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} \alpha_{ijkl} S_{kl}$$ in a simpler form as $$\tau_i = \sum_{j=1}^{6} c_{ij} \varepsilon_j \quad i = 1, \ldots, 6$$

- This form allows for 6 x 6 = 36 independent elastic constants but many materials can be described by significantly fewer constants

Constants c_{ij} are called the elastic stiffness coefficients and have dimensions of N/m^2

Linear elastic response

- The simplest case is that of homogeneous, **isotropic materials.** They look the same regardless of the orientation
- This corresponds to neglecting all information on the spatial arrangement of the atoms inside the material
- The elastic moduli for these types of material have the form:

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} \quad \text{where} \quad c_{44} = (c_{11} - c_{12})/2$$

- These constants are also referred to as the Lamé constants λ and μ and are defined as:

$$\lambda = c_{12}, \quad \mu = c_{44} = (c_{11} - c_{12})/2$$

so there are only two independent elastic constants

Linear elastic response

- The elastic properties of isotropic materials are most often reported in terms of the Young's modulus E, Poisson ratio ν and shear modulus G

- These quantities are given in terms of the Lamé constants by the relations:

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \quad G = \frac{1}{\mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}$$

or in terms of the elastic moduli:

$$E = \frac{(c_{11} - c_{12})(c_{11} + 2c_{12})}{(c_{11} + c_{12})}, \quad G = \frac{1}{c_{12}}, \quad \nu = \frac{c_{12}}{c_{11} + c_{12}}$$

Elastic moduli of some materials

Material	Young's modulus E (GPa)	Shear modulus G (GPa)	Poisson's ratio ν
Aluminum	70	26	0.33
Brass	96-110	36-41	0.34
Steel	190-210	75-80	0.27-0.3
Silicon	150	80	0.17
SiO_2	43-77	30	0.17
Si_3N_4	325	127	0.24